Fuzzy Inference of Soil Patterns: Implications for Watershed Modeling

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A fuzzy inference scheme for inferring and representing detailed soil spatial information is first reviewed. This scheme consists of two major components: a fuzzy logic-based model (called a similarity model) and a set of inference techniques. Under the similarity model the soil landscape is perceived as a continuum in both its parameter space and geographic space so that detailed spatial variation of soil information can be represented. A set of inference techniques were used to populate this similarity model by combining the knowledge of soil experts with data on soil formative environment. The provision of this detailed soil spatial information represented under this similarity model has significant implications for watershed modeling, particularly for solute transport modeling in the vadose zone. It was found that the characterization of a key soil parameter (solum depth) based on the soil spatial information derived from the fuzzy inference scheme differs significantly from that derived from the conventional soil map for watershed modeling using a lumped-parameter approach. It was also found that the provision of this detailed soil spatial information allows the realistic characterization of the spatial co-variation of landscape parameters for distributed modeling at the watershed scale. Specifically, the within-land-unit variation of solum depth is more realistically characterized using the inferred soil spatial information than using the conventional soil map.

INTRODUCTION

The transport of pollutants in the vadose zone largely depends on the soil properties (e.g., hydraulic conductivity, water holding capacity, texture, bulk density) influencing the hydraulic characteristics of soil. Information on spatial variation of these and many other soil properties is required for modeling the spatial variability of non-point source (NPS) pollutants and their transport in the vadose zone at watershed scales within the context of a GIS [Burrough, 1996; Corwin, 1996; Corwin et al., 1997; Ellsworth, 1996; Jury, 1996]. Soil maps currently are the major existing source for deriving information on the spatial variation of these soil properties [Lytle, 1993]. Yet, the soil spatial information derived from conventional soil maps has been found inadequate for most modeling activities at the mesoscale watershed level [Band and Moore, 1995; Zhu, 1997].

This inadequacy results from the following three factors. First, some of the model-sensitive soil properties, such as the soil moisture characteristic curve, one-third bar water content, and saturated hydraulic conductivity, are not reported in current Natural Resources Conservation Service (NRCS) soil databases [Nielsen et al., 1996]. Second, soil maps seldom provide information about the spatial distribution of soil properties at the resolution desired (both at spatial and attribute levels) for GIS-based pollution assessment models in the vadose zone [Nielsen et al., 1996] and other hydro-ecological modeling at the watershed scale [Band and Moore, 1995; Zhu, 1997]. Third, soil map spatial information is found incompatible with data derived from digital terrain analysis and remote sensing techniques in both the spatial and attribute domains [Zhu, 1997]. This
incompatibility prevents the spatial co-variation of landscape parameters from being realistically characterized. This co-variation is highly desirable for us to understand and model many hydro-ecological processes in the vadose zone at the watershed scale.

This paper reviews a soil similarity scheme that addresses the second and third factors. The specifics of this scheme have been published elsewhere [Zhu, 1997; Zhu et al., 1996; Zhu et al., 1997]. An overview is presented to provide a context by which detailed soil spatial information can be derived. This paper also discusses the limitations of the conventional discrete spatial model for soil mapping, addresses the implications of detailed soil spatial information for landscape parameterization, and examines the implications of the detailed soil spatial information represented under this scheme for modeling NPS pollutants at the watershed scale.

SOIL MAPPING AND RECENT EFFORTS

Limitations of Conventional Soil Mapping

Conventional soil mapping is based on the “discrete spatial data model” by Bregt [1992], the “object model” by Goodchild [1992], or the “area-class map” concept by Mark and Csillag [1989]. Under the discrete spatial data model, soil spatial distribution is represented through the delineation of soil polygons with each polygon depicting the spatial extent of a particular soil type (class) or a group of commonly found classes (mixed-class mapping units). This model of soil mapping has two limitations [Zhu, 1997]. The first is that the polygons represent the distribution of a set of prescribed soil classes, not individual soils in the field which are often different from the prototypes of the prescribed soil classes. In order to be mapped, individual soils in the field must be classified based on some classification scheme. Each individual soil is often assigned to one and only one class. Once assigned to a class the local soil is said to have the typical properties of that class. This means that the varying soil continuum is now represented by a few distinct and discrete soil classes. The domain of a soil property is in fact approximated by some typical values of soil classes, which are often discrete. This generalization of the entire domain of a soil property into a few typical values is referred to as generalization of soils in the parameter domain. This generalization coupled with the polygon-based mapping practice forces soil spatial variation to be represented as a step function, which means that soil variation only occurs at the boundaries of soil polygons and everything within a given soil polygon is the same. This representation of soil spatial variation is not realistic since soil properties often vary gradually over space [Burrough, 1993; Jury, 1985; Mark and Csillag, 1989; Warrick and Neilsen, 1980; White, 1988].

The second limitation is that only “soil bodies” larger than a certain size can be shown on the map. This size, termed minimum mapping size, is often a function of the smallest polygon which can be represented on the map at a given scale. “Soil bodies” smaller than this size are either ignored or merged into the larger enclosing soil bodies. The existence of these smaller “soil bodies” may or may not be reported in the map legend. In any case, the spatial extent of these smaller soil bodies is not shown in the map. This filtering of small soil bodies due to the limitation of cartographic techniques is called generalization of soils in the spatial domain. This spatial generalization of soils can be very significant and the soil bodies that are filtered out can range from a few hectares on some large-scale (small area) maps to hundreds of hectares on some small-scale (large area) maps. These patches of different soils could provide useful insights on some important environmental niches. For example, these niches could be critical to studies on preferential flow paths through which water and chemicals move much more quickly than in the soil as a whole [Jury, 1996].

The generalization of soils in both the parameter and the spatial domains through conventional soil mapping also makes the soil spatial information derived from such maps incompatible with data derived from digital terrain analyses and remote sensing techniques (referred to herein as other detailed environmental data). Zhu [1997] reported the incompatibilities to be of two kinds: attribute incompatibility and spatial incompatibility. The former refers to the difference between the attribute resolution used to describe the values of soil properties and that used to describe the values of other detailed environmental data. Attribute resolution here refers to the degree of detail in describing an attribute value. On a conventional soil map, changes of soil attribute values between neighboring locations (Figure 1a) were either completely ignored (Figure 1b) or described as the difference between two different soil classes (Figure 1c). On the other hand, data derived from digital terrain analysis such as elevation and slope gradient normally retain the subtle differences in the attribute values between neighboring locations. Spatial incompatibility refers to the difference in spatial resolution between conventional soil maps and data derived from digital terrain analysis and remote sensing techniques. Spatial resolution as used in this context refers to the level of spatial detail at which the spatial variation is represented. It should be noted that simply converting a soil map into a raster layer and reducing the pixel size do not at all increase the spatial resolution of the data layer since the process adds no
results from a variety of large-scale (small area) solute transport models, but could also jeopardize our ability to understand many pollutant transport processes in the vadose zone.

**Recent Efforts**

The two generalizations in conventional soil mapping are due to the limitations imposed by the discrete spatial data model and cartographic techniques. It may be true that local soil experts understand the detailed variation of soil over space and the inclusion of different soil bodies in soil mapping units, but the discrete spatial data model and cartographic techniques used in soil mapping limit their ability to fully express their understanding of soil spatial variation. Many researchers have thus explored other means of characterizing soil landscape. These studies have led to the development of three major groups of approaches for quantifying and representing soil spatial variations: approaches on modifying existing soil maps; statistical/geostatistical approaches; and fuzzy logic-based approaches.

Several researchers have attempted to incorporate spatial variability into existing soil maps. Maclean et al. [1993] used Shannon's measure of entropy to determine interpretive variabilities of soil map units. The derived interpretive variabilities were represented as soil variability diagrams. These diagrams express the degree of variability of soils within the respective soil mapping units as non-variable, moderately variable, and highly variable. The method requires soil samples on transects across the given soil mapping unit. These soil samples are often not available for existing soil maps. Another limitation of this method is that it does not present the spatial distribution of variability within the mapping unit. Ferguson and Hergarten [1998, this publication] employ geostatistical tools and other sampling methods, including yield mapping and remote sensing, to provide finer detail in the delineation of soil mapping units. Yost et al. [1998, this publication] illustrate six methods of including spatial information into existing soils databases. In an effort to incorporate field variability into existing soils databases, Rogowski [1996] proposed an overlay method that combines measured and published delineations of soil properties into an overlay with the properties of both. The aforementioned efforts certainly add value to soil maps, but the underlying spatial discrete model and the cartographic techniques used in producing these soil maps still pose challenges for portraying soil as a spatial continuum.

The statistical approaches first extract the relationships between soil properties and other landscape factors from point samples and then use the relationships with the data on landscape factors in a GIS to predict the soil properties.
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over an area [Moore et al., 1993; Gessler et al., 1995]. The geostatistical approaches make use of the spatial autocorrelation of soil properties in the field-sampled data sets for interpolating the soil property values at unknown sites [for example, Bierkens and Burrough, 1993; Burgess and Webster, 1980; Journel, 1986; Loague, 1992; McBride and Webster, 1986; Webster, 1991; Webster and Oliver, 1989; Zhang et al., 1995]. These statistical/geostatistical techniques assume that the relationships (including spatial autocorrelations) are static over space and require a great deal of field data to extract these relationships. These techniques may have limited usage for complex terrain where pedogenesis arises in a complex manner and the stationarity assumptions of these techniques may not be met.

The fuzzy logic-based approaches employ fuzzy logic in the soil/land classification process [for example, Burrough, 1989; Burrough et al., 1992; Burrough et al., 1997; De Gruijter et al., 1997; Lagacherie et al., 1997; McBride and De Gruijter, 1992; Odeh et al., 1992a; see McBride and Odeh, 1997 for a comprehensive overview of this field]. Under fuzzy logic, soils in fields can be assigned to more than one class with varying degrees (memories) of class assignment so that the soil gradation in the parameter domain can be described using these membership values. Fuzzy classification of soils often employs the unsupervised classification strategy, which is the partitioning of observations in multivariate space into relatively stable naturally occurring groups. These groups do not necessarily relate to any of the existing soil taxonomic classes. Odeh et al. [1992b] employed a kriging technique to create isarithmic maps of membership distribution of some fuzzy soil classes derived from a fuzzy-c-means (FCM) classification procedure. This is one of the earliest attempts to represent soil landscape as a continuum in geographic domain. However, the kriging technique requires a great deal of field data and the stationarity assumption. This often presents challenges for deriving fuzzy membership maps over large areas, particularly over complex terrains.

Although fuzzy classification of soils provides a means for modeling the soil gradation in the parameter domain, there are still two important questions regarding the utility of fuzzy logic for representing soil spatial information. First, how would one derive fuzzy membership maps over large areas without the enormous cost of collecting field data? Second, how would one use the fuzzy membership maps for deriving detailed soil property maps? A soil similarity scheme is reviewed, which attempts to address these two questions, and then the implications of the detailed soil spatial information derived under the similarity scheme are examined for modeling NPS pollutant transport in the vadose zone.

THE SOIL SIMILARITY SCHEME

The Similarity Model

Zhu [1997] developed a similarity scheme for overcoming the two generalizations in representing soil spatial information. The scheme consists of two components: a similarity model and a set of techniques for populating such a model. The similarity model is composed of two parts: the raster representation of geographic objects in the spatial domain and the similarity representation of these objects in the parameter domain. In raster GIS data modeling, spatial objects are represented by many small squares (pixels). The pixel size can be very small; it is often 30 meters on each side, although much finer pixel sizes are possible. With raster representation, spatial generalization in producing soil spatial information can be largely reduced and spatial details of soil variation can be represented at the resolution compatible with the detailed terrain and remotely sensed data. This would resolve the spatial incompatibility between soil spatial information and these detailed environmental data [Zhu, 1997].

The similarity representation of geographic objects in the parameter domain is based on fuzzy logic, which allows an object at a given location (pixel) to be represented as intermediate to a set of prescribed classes (often existing taxonomic classes). In this way, an entity at a given pixel is not assigned to one and only one class. It is, in fact, allowed to bear a partial membership in each of the prescribed classes. Each of these membership values (fuzzy memberships) can be regarded as a similarity measure between the entity and the typical case of the given class. All of these fuzzy memberships (similarity values) are retained in this similarity representation, which forms an n-element vector (similarity vector, or fuzzy membership vector), \( S_{ij}^{k} \) \( (S_{ij}^{1}, S_{ij}^{2}, ..., S_{ij}^{k}, ..., S_{ij}^{n}) \), where \( n \) is the number of prescribed classes and the \( k \)th element \( S_{ij}^{k} \) in the vector represents the similarity value between the entity at pixel \( (i,j) \) and class \( k \). With this similarity representation, a local entity such as a soil pedon at a given location is no longer necessarily approximated by the typical case of a particular class but can be represented as an inter-grade to the set of prescribed classes. This model of representation, which allows the local soil to take property values intermediate to the typical values of the prescribed classes, overcomes the attribute incompatibility between the soil spatial information represented under the similarity model and the other detailed environmental data.
Populating the Similarity Model

Under the similarity model, soil spatial information over an area is represented as an array of pixels with the soil at each pixel being described by a soil similarity vector. The next question is how to derive a soil similarity vector (fuzzy membership vector) for the soil at each pixel. There are many possible approaches to the population of this similarity model such as supervised or unsupervised fuzzy classifications [Bezdek et al., 1984; McBratney and De Gruijter, 1992; Odeh et al., 1992a; Wang, 1990], generalized linear models, and neural networks [Rumelhart et al., 1986]. However, the usefulness of these approaches for populating the similarity model are yet to be examined. Zhu et al. [1996] and Zhu and Band [1994] developed a knowledge-based strategy for deriving soil similarity vectors based on the classic concept of Jenny [1980, 1994]. This concept contends that relationships exist between soils and their formative environment. Though the specifics of this knowledge-based method are beyond the scope of this paper, in general, Zhu et al. [1996] and Zhu and Band [1994] used GIS techniques to characterize and represent soil formative environments and employed a set of knowledge acquisition techniques [Zhu, 1995] to capture the knowledge of relationships between soils and their formative environments from local soil scientists (Figure 2). The information on soil formative environments and the knowledge of soil-environment relationships were then integrated through a set of fuzzy inference techniques (fuzzy inference engine) to populate the soil similarity vectors (see [Zhu and Band, 1994] for details on the inference engine).

Figure 3 illustrates the general process of fuzzy soil inference for the Lubrecht watershed (Figure 4, see [Zhu et al., 1996] for details on the study area). Six environmental variables (elevation, parent material, canopy coverage, slope aspect, slope gradient, and surface profile curvature) were used to characterize soil formative environment [Zhu and Band, 1994]. Twelve soil series served as the central concept for fuzzy soil classes [Zhu et al., 1996]. These soil series are formed from three parent materials: granite (with soil series Ambrant, Elkner, Ovando, and Rochester), Belt
Methodology

Knowledge Acquisition

(GIS Techniques)

(Fuzzy Inference Engine)

(Similarity Representation)

Figure 3. Fuzzy soil inference process for the Lubrecht study area.

(with soil series Evaro, Sharrott, Tevis, Winkler, and Winkler Cool), and limestone (with soil series Repp, Trapps, and Whitore). The knowledge of soil-environment relationships was derived from a local certified soil scientist, Barry Dutton, using a knowledge elicitation approach [Zhu, 1995].

A soil similarity vector for each pixel in the GIS database was produced using a fuzzy inference engine (Figure 3) [Zhu and Band, 1994]. At a given pixel, the inference engine retrieves data on a set of environmental conditions from the GIS database for that pixel and combines them with the soil-environment relationships to derive the similarity vector for that pixel. This process was repeated for every pixel in the database and a similarity representation of soils was then created for the area (see [Zhu and Band, 1994] for details on the inference process).

The soil similarity vectors for a few selected field sites on the granite materials in the Lubrecht area are shown in Table 1. Although the fuzzy membership values in these vector range from 0 to 100, the sum of these values within a given vector may or may not equal unity (100) when existing soil classes are used as the central concepts of fuzzy classes [Zhu, 1997]. It is worth pointing out that a soil at a given point is represented by the entire vector and every fuzzy membership value in the vector is important since the membership values together define the uniqueness of the soil at that location. For example, Site 91_03 and Site Lub06_02 both bear the highest memberships to soil series Elkner. However, the membership distribution in the vector for Site 91_03 is very different from Site Lub06_02. It is this difference in

<table>
<thead>
<tr>
<th>Site ID</th>
<th>Ambrant</th>
<th>Rochester</th>
<th>Elkner</th>
<th>Ovando</th>
</tr>
</thead>
<tbody>
<tr>
<td>lb06_02</td>
<td>9.41</td>
<td>0.00</td>
<td>53.79</td>
<td>52.80</td>
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<td>29.37</td>
<td>29.23</td>
<td>16.25</td>
<td>45.81</td>
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<td>30.36</td>
<td>0.19</td>
<td>1.85</td>
</tr>
<tr>
<td>lb10_03</td>
<td>7.63</td>
<td>1.55</td>
<td>0.00</td>
<td>47.49</td>
</tr>
<tr>
<td>91_03</td>
<td>16.54</td>
<td>14.47</td>
<td>69.23</td>
<td>39.79</td>
</tr>
</tbody>
</table>
membership distribution which allows us to infer that the soil at Site 91_03 is different from that at Site Lub06_02 and therefore allows us to expect the soil property values at these two sites to be different [Zhu et al., 1997].

**Deriving Detailed Soil Property Maps**

Zhu et al. [1997] illustrate the use of soil similarity vectors for deriving soil property maps. In their illustration, they used the following linear and additive weighting function to estimate the soil A-horizon depth for every pixel in the database:

$$D_y = \frac{\sum_{k=1}^{n} S_y^k \cdot D^k}{\sum_{k=1}^{n} S_y^k}$$  \hspace{1cm} (1)

where $D_y$ is the soil A-horizon depth at site $(i, j)$, $S_y^k$ is the similarity measure between the soil at site $(i, j)$ and soil series $k$, $D^k$ is the prescribed soil A-horizon depth of soil series $k$, and $n$ is the total number of prescribed soil series in the area.

The soil A-horizon depth image derived using Eq. (1) and that from the conventional soil map for the Lubrecht study area are shown in Figure 5. The difference in spatial variation of soil A-horizon depth portrayed by the similarity scheme (Figure 5a) from that by the conventional soil map (Figure 5b) is obvious. Although both images show that A-horizon depths are shallower on south facing slopes and at low elevations than on north facing slopes and high elevations, the major difference is how the spatial variation of soil A-horizon depth is portrayed in these two images. The depth image based on the similarity scheme shows a spatially continuous pattern of A-horizon depth over the area while the image derived
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from the soil map inherits the exact spatial pattern of the soil map, on which the spatial variation of A-horizon depth was discretized into distinct and discrete spatial units and perceived as a step-function.

The other difference between the two images (Figure 5) is the spatial details of soil A-horizon depth. The depth image based on the similarity scheme shows the A-horizon depth at much greater spatial detail than does the conventional soil map. For this mountainous terrain with a semi-humid to semi-arid climate [Nimlos, 1986], soil A-horizon depths are expected to be highly variable over space since subtle changes in exposure could result in significant changes in local moisture conditions, which in turn impact the development of soils and the A-horizon depths at these locations. For example, the side slopes of small draws on a major south facing slope would face away from direct south and would have better moisture conditions than those areas facing direct south. The soils on these side slopes are expected to be better developed and the soil A-horizon depths are expected to be deeper than in areas facing directly south. These differences in A-horizon depth between the small draws and the surrounding areas on the south-facing slope of the North Fork of Elk Creek are observable in Figure 5a. However, the depth image based on the soil map fails to show these differences. Based on the soil map, soil A-horizon depths were to be homogeneous within each of the soil polygons and change only at the boundaries of these polygons. This apparent homogeneity of soil A-horizon depth over the study area is not realistic for this mountainous terrain.

The attribute resolution is much greater in the depth image from the similarity scheme than in the conventional soil map. Figure 6 shows the scatter plots between the observed depths in the field and the depths both from the similarity scheme and from the soil map at 33 field sites. The depths from the similarity scheme correspond, as measured by the coefficient, to the observed depths better than those derived from the soil map. Further examination of Figure 6 also reveals that the depths from the similarity scheme for these sites are continuous variables. On the other hand, the depths derived from the soil map are limited to the values of the prescribed soil series.

**IMPLICATIONS FOR WATERSHED MODELING**

An important part of this paper is to examine the implications of the detailed soil spatial information derived from the similarity scheme for modeling NPS pollutants in the vadose zone at the watershed scale. This discussion is limited to the implications for two different kinds of models: lumped-parameter and distributed models [Chow et al., 1988].

The transport of NPS pollutants in the vadose zone depends heavily on two key soil hydraulic properties (soil-water holding capacity and soil-water transmissivity). Soil-water transmissivity is defined as the depth-integrated saturated hydraulic conductivity. Both soil-water holding capacity and transmissivity are related to the rooting zone depth. In this discussion, the solun depth is used to approximate the rooting zone depth. Although other physical soil properties also influence these soil hydraulic properties, only spatial information on solun depth is used to illustrate how different representations of spatial variation of solun depth would impact model parameterization for both distributed and lumped-parameter models. With

**Figure 6.** Scatter plots of depths for the 33 field sites. (a) Observed vs. similarity scheme; (b) Observed vs. soil map.
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![Image](https://via.placeholder.com/150)  
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the understanding that various definitions exist, solum depth is defined here as the depth of soil to the bottom of B horizon.

Two representations of solum depth were computed for the Lubrecht study area. The first one was based on the similarity scheme (Figure 7a). The solum depth \( D_{ij} \) at each pixel \((i,j)\) was calculated using Eq. (1) with \( D^k \) being the typical solum depth of soil category \( k \). The other representation of solum depth (Figure 7b) was derived from the soil map of the study area by assigning each pixel the typical solum depth of the soil class to which the pixel is assigned.

The spatial pattern shown in Figure 7a follows that of the \( A \)-horizon depth (Figure 5a). It is reasonable to believe that an area with a deep \( A \)-horizon would have a deep profile since human disturbance in this area has not been significant [Nimlos, 1986]. Also, the spatial pattern of solum depth matches the moisture regime in this area well. Since the area is in a semi-humid to semi-arid region of western Montana, the moisture condition on north-facing slopes at high elevations is better than on south-facing slopes at lower elevations. Soils on north faces at high elevations are expected to be deeper than those on the south faces at lower elevations. Although the solum depth data layer has not yet been explicitly validated, it is expected that the spatial pattern of solum depth based on the similarity scheme is more realistic than that portrayed in the conventional soil map.

Implications for Lumped-Parameter Models

A deterministic lumped-parameter model is an abstract representation of spatial features in which properties are averaged over a watershed, stream segment or slope facet [Maidment, 1993]. In other words, the process over an area is modeled as a single point in space without dimensions and the system is spatially averaged [Chow et al., 1988]. Internal spatial variability is ignored and modeled process and model parameters are assumed to be uniform within the given spatial unit (e.g., a slope facet, catchment or an entire watershed). The main objective of lumped-parameter models is to simplify the model parameterization over space and to produce an unbiased model result to represent the average condition of the modeled process over the entire area.

The idea of producing an unbiased model output using the lumped-parameter approach is based on the assumption that the result of a lumped-parameter model over an area can be expressed as a Taylor Series expansion about the mean values of the parameters as below [Band et al., 1991]:

\[
P(x) = p(\bar{a}_1, \bar{a}_2, ..., \bar{a}_m) + \sum_{i=1}^{m} \frac{\partial p(\bar{a}_1, \bar{a}_2, ..., \bar{a}_m)}{\partial \bar{a}_i} (a_i - \bar{a}_i) + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=i+1}^{m} \frac{\partial^2 p(\bar{a}_1, \bar{a}_2, ..., \bar{a}_m)}{\partial \bar{a}_i \partial \bar{a}_j} (a_i - \bar{a}_i)(a_j - \bar{a}_j) + ...
\]

where \( P \) is the model process, \( P(x) \) is the model output for spatial unit \( x \) (where \( x \) can be a pixel, slope segment or entire watershed), and \( \alpha_i \) are the model parameters specific to the unit \( x \). The first term in the right hand side of Eq. (2) gives the result of using the mean values of the model parameters over unit \( x \) and the next two terms add the deviations to that result caused by the deviation of each parameter from its mean value, and covariance between

![Solum (A+B) Depth Based on the Similarity Scheme](image1)

![Solum (A+B) Depth Derived from the Soil Map](image2)

Figure 7. Images of solum depth (depth of horizon \( A \) + depth of horizon \( B \)) for the Lubrecht area. (a) Based on the similarity scheme; (b) From the soil map.
the understanding that various definitions exist, solum depth is defined here as the depth of soil to the bottom of B horizon.

Two representations of solum depth were computed for the Lubrecht study area. The first one was based on the similarity scheme (Figure 7a). The solum depth ($D_{ij}$) at each pixel $(i,j)$ was calculated using Eq. (1) with $D_k^k$ being the typical solum depth of soil category $k$. The other representation of solum depth (Figure 7b) was derived from the soil map of the study area by assigning each pixel the typical solum depth of the soil class to which the pixel is assigned.

The spatial pattern shown in Figure 7a follows that of the $A$-horizon depth (Figure 5a). It is reasonable to believe that an area with a deep $A$-horizon would have a deep profile since human disturbance in this area has not been significant [Nimlos, 1986]. Also, the spatial pattern of solum depth matches the moisture regime in this area well. Since the area is in a semi-humid to semi-arid region of western Montana, the moisture condition on north-facing slopes at high elevations is better than on south-facing slopes at lower elevations. Soils on north faces at high elevations are expected to be deeper than those on the south faces at lower elevations. Although the solum depth data layer has not yet been explicitly validated, it is expected that the spatial pattern of solum depth based on the similarity scheme is more realistic than that portrayed in the conventional soil map.

**Implications for Lumped-Parameter Models**

A deterministic lumped-parameter model is an abstract representation of spatial features in which properties are averaged over a watershed, stream segment or slope facet [Maïmènt, 1993]. In other words, the process over an area is modeled as a single point in space without dimensions and the system is spatially averaged [Chow et al., 1988]. Internal spatial variability is ignored and modeled process and model parameters are assumed to be uniform within the given spatial unit (e.g., a slope facet, catchment or an entire watershed). The main objective of lumped-parameter models is to simplify the model parameterization over space and to produce an unbiased model result to represent the average condition of the modeled process over the entire area.

The idea of producing an unbiased model output using the lumped-parameter approach is based on the assumption that the result of a lumped-parameter model over an area can be expressed as a Taylor Series expansion about the mean values of the parameters as below [Band et al., 1991]:

$$P(x) = p(\bar{a}, a_1, a_2, \ldots, a_n) + \sum_{i=1}^{n} \frac{\partial p(\bar{a}, a_1, a_2, \ldots, a_n)}{\partial a_i} (a_i - \bar{a}_i)$$

$$+ \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 p(\bar{a}, a_1, a_2, \ldots, a_n)}{\partial a_i \partial a_j} (a_i - \bar{a}_i)(a_j - \bar{a}_j)$$

$$+ \ldots$$

where $P$ is the model process, $p(x)$ is the model output for spatial unit $x$ (where $x$ can be a pixel, slope segment or entire watershed), and $a_i$ are the model parameters specific to the unit $x$. The first term in the right hand side of Eq. (2) gives the result of using the mean values of the model parameters over unit $x$ and the next two terms add the deviations to that result caused by the deviation of each parameter from its mean value, and covariance between

![Solum (A+B) Depth Based on the Similarity Scheme](image1)

![Solum (A+B) Depth Derived from the Soil Map](image2)

**Figure 7.** Images of solum depth (depth of horizon $A$ + depth of horizon $B$) for the Lubrecht area. (a) Based on the similarity scheme; (b) From the soil map.
the parameters. If there were negligible variance of the model parameter over spatial unit \( x \) relative to the model sensitivity, the latter two terms would tend to be zero, which would give us Eq. (3) [Band et al., 1991]:

\[
P(x) = p(\bar{\alpha}_1, \bar{\alpha}_2, \ldots, \bar{\alpha}_i, \ldots, \bar{\alpha}_n) \tag{3}
\]

which means that the model result over an area \( x \) can be approximated by the model result using the mean values of the model parameters over unit \( x \) if the model is linear over the range of \( \alpha_0 \), or the variances and covariances are small enough. Whether or not the output from a lumped-parameter model is biased depends on two important aspects: the degree to which mean values of the model parameters are accurately parameterized over an area and the extent to which partitioning of the landscape produces landscape units with sufficiently small internal variances and covariances of the model parameters.

The impacts of different representations of soil spatial information on lumped-parameter models are examined using the Lubrecht site. Let us consider the modeled process as pollutant generation. However, in this case the interest is in the total pollution generated from each of the partitions without concern for internal variability. One of the model parameters is the soil hydraulic property, which is approximated by the solum depth data. The Lubrecht watershed was partitioned into 42 partitions\(^1\) using a method of landscape partitioning from digital elevation data based on accumulative upslope drainage area [Band, 1989]. There are two solum depth data: one based on the similarity scheme and the other derived from the conventional soil map. Each representation of solum depth was then overlaid onto these partitions to calculate the mean solum depth and to compute the variability of solum depth for each of the partitions. Thus, for each partition there are two mean solum depths and two measures of spatial variability of solum depth.

The differences between the two solum depth representations for mean solum depths within these 39 partitions are shown in Table 2. The differences for 14 out of 39 partitions are greater than 30% of the mean solum depth computed from the soil map. The largest difference (for Partition 27) is about 128%. The average difference in mean solum depth over all partitions is about 24% (Table 2). A student-\( t \) test (Table 3) was used to determine whether the differences in mean solum depth between the

| Table 2: Mean Solum Depths Based on Different Representations of Soil Information. |
|-----------------|-----------------|-----------------|-----------------|
| Hillslope ID | Mean (the Scheme) | Mean (Soil Map) | Difference (Absolute) | Difference in Proportion\(^2\) |
| 1 | 109.76 | 76.00 | 33.76 | 0.44 |
| 2 | 99.57 | 88.94 | 10.63 | 0.12 |
| 4 | 82.50 | 76.00 | 6.50 | 0.09 |
| 5 | 106.43 | 85.55 | 20.88 | 0.24 |
| 6 | 111.31 | 118.50 | 7.19 | 0.06 |
| 7 | 107.67 | 95.24 | 12.43 | 0.13 |
| 8 | 78.35 | 76.00 | 2.35 | 0.03 |
| 9 | 106.73 | 81.60 | 25.13 | 0.31 |
| 10 | 122.80 | 93.09 | 29.71 | 0.32 |
| 11 | 107.50 | 97.51 | 9.99 | 0.10 |
| 12 | 76.26 | 66.73 | 9.54 | 0.14 |
| 13 | 89.25 | 49.35 | 39.90 | 0.81 |
| 14 | 100.18 | 78.80 | 21.38 | 0.27 |
| 15 | 97.57 | 86.25 | 11.32 | 0.13 |
| 16 | 106.27 | 91.30 | 14.97 | 0.16 |
| 17 | 78.37 | 66.97 | 11.40 | 0.17 |
| 18 | 89.83 | 96.07 | 6.24 | 0.06 |
| 19 | 101.00 | 93.83 | 5.17 | 0.05 |
| 21 | 91.11 | 84.97 | 6.14 | 0.07 |
| 22 | 58.87 | 39.77 | 19.10 | 0.48 |
| 23 | 98.75 | 97.74 | 1.02 | 0.01 |
| 24 | 61.91 | 46.10 | 15.82 | 0.34 |
| 25 | 92.67 | 93.95 | 1.28 | 0.01 |
| 26 | 65.83 | 62.14 | 3.70 | 0.06 |
| 27 | 82.37 | 56.20 | 66.17 | 1.28 |
| 28 | 63.73 | 52.46 | 11.28 | 0.21 |
| 29 | 90.95 | 85.11 | 5.84 | 0.07 |
| 30 | 60.58 | 44.53 | 16.05 | 0.36 |
| 31 | 88.01 | 93.44 | 5.43 | 0.06 |
| 32 | 65.47 | 106.59 | 41.12 | 0.39 |
| 33 | 107.15 | 76.00 | 31.15 | 0.41 |
| 34 | 68.87 | 86.78 | 17.92 | 0.21 |
| 35 | 127.72 | 98.13 | 29.59 | 0.30 |
| 36 | 118.99 | 88.60 | 30.39 | 0.34 |
| 37 | 119.21 | 78.37 | 40.84 | 0.52 |
| 38 | 118.86 | 117.89 | 0.97 | 0.01 |
| 39 | 136.29 | 148.87 | 12.58 | 0.08 |
| 41 | 82.16 | 79.41 | 2.76 | 0.03 |
| 42 | 59.48 | 38.18 | 21.30 | 0.56 |
| Mean | 93.09 | 81.41 | 16.38 | 0.24 |

\(^1\) Only 39 partitions are reported here since the other three partitions fall on the stream areas which were not included in this study.

\(^2\) Proportion is based on the mean computed from the soil map.
Table 3: Statistics about the Average of Differences in Mean Solum Depth.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Values</th>
<th>T-test</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>16.38</td>
<td>Std. Error</td>
<td>2.03</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>12.67</td>
<td>T (calculated)</td>
<td>8.06</td>
</tr>
<tr>
<td>N</td>
<td>39</td>
<td>df</td>
<td>38</td>
</tr>
</tbody>
</table>

two representations are statistically significant. The critical t value at 95% confidence with degree of freedom of 38 is 1.69, which is significantly less than the calculated t of 8.08. It can be concluded that the two groups of mean solum depths are statistically different from each other. If the total amount of pollutants generated from each of the partitions was computed according to Eq. (3) and the model is sufficiently sensitive to this parameter, then the amount of pollutants computed based on the mean solum depths from the similarity scheme would be different from that computed based on the mean solum depths from the soil map. If the actual implemented form of Eq. (3) is a good approximation of reality, then the amount computed using the solum depth data based on the similarity scheme would be a better estimation of the actual amount.

The variability of solum depth within Partition 24 of the Lubrecht area (Figure 8) for each of the representations of soil spatial information is presented in Figure 9. Figure 9a shows the frequency distribution of different solum depths based on the similarity scheme, which demonstrates a variability of solum depth within that partition in both the range of values and the frequency distribution. However, Figure 9b, which shows the frequency distribution of different solum depths based on the soil map for the same partition, depicts a fairly homogenous distribution of solum depth. According to Figure 9b, there are only three different solum depths within the partition, and the range of these depths is much narrower than portrayed in Figure 9a. Although each partition is expected to have a reasonable homogeneity in its aspect, subtle changes in slope exposure could induce significant differences in moisture regime in this semi-humid to semi-arid mountainous terrain. Therefore, it is expected that the variability of solum depth would still be substantial. If the variability of solum depth within each partition were used to adjust the amount computed from Eq. (3), then the within-partition variability portrayed by the soil map would have less influence than the within-partition variability depicted by the similarity scheme.

Implications for Distributed Models

A distributed model is a model in which the modeled process or phenomenon is explicitly calculated as a function of location in space [Maidment, 1993]. In other words, the output of the model process at one location will be one of the inputs for a neighboring location. For distributed modeling, the interest is not just in the overall outcome of the modeled process over an entire watershed, but more importantly in the spatial variation of the modeled process within the watershed. This requires a realistic representation of model parameters at every location over the modeled watershed so that the spatial co-variation of these model parameters can be characterized. It is preferable that all model parameters are represented at the same spatial resolution used in modeling the physical process so that small but significant environmental niches can be taken into account. This detailed representation of spatial variation allows spatial variation of the modeled process to be realistically simulated. If the spatial variation of one of the model parameters is represented at a much coarser resolution than the resolution used in modeling, then the spatial co-variation of model parameters would be biased. Thus, the model output may be biased, particularly over the small but important environmental niches, since the coarse resolution of an important parameter cannot capture the characteristics of these niches. The ability to understand the modeled process and its spatial variation over the watershed would potentially be handicapped by biased model results.

The impact of different representations of spatial information on detailed distributed modeling can be further illustrated through the following hypothetical example. Suppose that the characterization of spatial variation of soil hydraulic properties (soil-water-holding capacity and soil-water transmissivity) were needed to model the spatial

![Partition 24](image)

Figure 8. Location of Partition 24 in the Lubrecht watershed.
Using Solum Depth Based on the Similarity Scheme

![Graph showing solum depth distribution](image)

(a)

Using Solum Depth From the Conventional Soil Map

![Graph showing solum depth distribution](image)

(b)

**Figure 9.** Variability of solum depth within Partition 24 (Figure 8). (a) Based on the similarity scheme; (b) Based on soil map.

variation in the transport of NPS pollutants in the vadose zone in the Lubrecht watershed. For simplicity, it is assumed here that pollutant transport depends on soil hydraulic properties and topographic positions. It is further assumed that solum depth data alone can be used to characterize soil hydraulic properties. If the solum depth derived from the soil map (Figure 7b) is used to characterize the soil hydraulic properties in the watershed, then areas within a soil polygon would be expected to have the same soil hydraulic properties as the typical hydraulic properties of the soil category to which the polygon is assigned. This means that the spatial variability in pollutant transport within a soil polygon such as the area labeled as A in Figure 7b would solely depend on the spatial variability in topographic positions. In this case, the solum depth information makes no contribution to the spatial variability in pollutant transport since there is no spatial variability in solum depth within that polygon. Thus, the model result about spatial variation in pollutant transport within each soil polygon would be biased. Also, the nature of pollutant transport would suddenly change at the boundaries of these soil polygons due to the sudden change in soil hydraulic properties prescribed by the soil map at these boundaries. This sudden change in pollutant transport is often unrealistic.

If the solum depth based on the similarity scheme were used to describe the spatial variation of soil hydraulic properties in the watershed, the spatial variation in pollutant transport would be different from that using the soil map. First, soil hydraulic properties are no longer divided into discrete and uniform regions, and are portrayed as spatially variable, which is more realistic in this mountainous terrain. This variability in soil hydraulic properties contributes to spatial variability in pollutant transport. For example, the spatial variability in pollutant transport over the area labeled as A in Figure 7b would no longer depend solely on the variability of other model parameters but also depend on the variability in soil hydraulic properties over the corresponding area, as portrayed in Figure 7a. Second, since the spatial variation of soil hydraulic properties is portrayed as more or less continuous over space, there will be no sudden changes in pollutant transport along artificial boundaries. Therefore the modeling of spatial variability in pollutant transport using the solum depth data based on the similarity scheme would be more realistic.

**SUMMARY AND CONCLUSIONS**

The similarity scheme is a means of quantifying and representing detailed soil spatial variation. Under the similarity model, soil spatial information is represented as an array of pixels, with each pixel covering a small amount of ground to capture the detailed spatial variation of soils. This raster representation reduces the cartographic generalization related to the scale of the map and the limitations of cartographic techniques. In the parameter domain, the model uses fuzzy logic for the representation of a soil at each pixel so that the local soil is not approximated by one and only one prescribed soil category. This fuzzy representation reduces the generalization of soil information in the parameter space. The similarity scheme provides a means for representing soil spatial information at greater detail at both the attribute and spatial level than the conventional soil map.
Data on solute depth were derived from the similarity representation of soil spatial information in the Lubrecht area. The data were compared with the solute depth data derived from a soil map of the area to study implications of detailed representation of soil shallow information for modeling NPS pollution in the vadose zone at the watershed scale. The implications were discussed in the context of two types of models: lumped-parameter and distributed models. For the lumped-parameter model, it was found that the mean solute depth for each partition based on the similarity scheme was very different from that based on the soil map. Since lumped-parameter models depend on the averaged conditions of model parameters over the given spatial unit, model output based on the similarity scheme is expected to be different from that based on the soil map. The examination also revealed that the within-partition variability of solute depth is greater using the solute depth data based on the similarity scheme than the soil map.

The detailed representation of spatial variation of solute depth by the similarity scheme allows the distributed models to simulate more realistically the spatial variation of modeled processes, which otherwise would be difficult to achieve with the use of soil solute depth data portrayed in the soil map.

It must be pointed out that the similarity model only provides the flexibility of representing detailed spatial information. The quality of the information depends on the way the similarity model is populated. The knowledge-based approach is only one of many ways for populating this similarity model. Efforts are underway to develop other means of populating such a similarity model. It should also be noted that the derivation of soil property values from the similarity representation described in Eq. (1) is for the purpose of this illustration. Other more sophisticated approaches of deriving soil property values from this similarity representation need to be explored.

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