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QUANTITATIVE ANALYSIS OF LAND SURFACE TOPOGRAPHY

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ABSTRACT

Land surface topography significantly affects the processes of runoff and erosion. A system which determines slope, aspect, and curvature in both the down-slope and across-slope directions is developed for an altitude matrix. Also, the upslope drainage area and maximum drainage distance are determined for every point within the altitude matrix. A FORTRAN 66 program performs the analysis.

KEY WORDS Terrain analysis Topography Hydrological modelling

INTRODUCTION

Geomorphologists and engineers have long recognized the importance of the study of land surfaces and the effects of topography on hydrologic and sedimentary processes. Slope, the most widely used topographic measurement, influences flow rates of water and sediment by controlling the rate of energy expenditure or stream power available to drive the flow. Aspect defines the slope direction and therefore the direction of flow. Knowledge of how aspect varies throughout a catchment provides the information necessary to determine what upslope land area contributes to the flow at any point in the catchment. Profile curvature, the rate of change of slope, affects flow acceleration and deceleration and therefore influences aggradation and degradation. The curvature of the land surface transverse to the slope direction, called planform curvature, influences flow convergence and divergence. A FORTRAN 66 program which analyses an altitude matrix for these four topographic indices plus upslope drainage area and the distance of the maximum drainage path has been developed. This program improves previous analyses because it objectively determines the upstream drainage area for every point within an altitude matrix.

PREVIOUS WORK

This discussion centres on topics of particular relevance to this paper. For a more in-depth review, Evans (1972) provides a thorough discussion on terrain analysis.

Rhind (1972) used patches of twelve elevations for fitting quadratics and cubic surfaces to aid in contour mapping, though no topographic parameters were evaluated from these surfaces. Young (1978) and Evans (1979) developed a method of topographic analysis yielding four topographic indices. Their analysis extends the methods presented by Tobler (1969) and Sharpnack and Akin (1969). The four topographic indices produced by Evans' system are slope, aspect, profile curvature and plan curvature. The analysis is based on a

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rectangular matrix of evenly spaced elevations covering the entire area in question. A 3 × 3 submatrix (Figure 1) is analysed repetitively throughout the rectangular altitude matrix. All topographic indices are related to the central point of the 3 × 3 submatrix. For the analysis, Evans chose a full quadratic surface given by the equation:

$$Z = Ax^2 + By^2 + Cxy + Dx + Ey + F \tag{1}$$

The nine elevations of the 3 × 3 submatrix are used to determine the six parameters, $A \ldots F$, of equation 1. This results in a surface which will not necessarily pass through the nine original elevations. The units of the topographic indices derived by Evans are: aspect, degrees (0–360); slope, degrees (0–90); profile and plan curvatures, degrees/100LU, where LU is the unit of length in which elevation is measured (i.e. LU = metres if Z is measured in metres). Evans (1980) suggests a wide variety of uses for topographic analysis. Of these suggestions, that of 'predicting the process rates and discharges from their relationships to attributes of land surface form' is of primary interest to the authors. For this purpose, however, upstream drainage area at a point is of great importance but is not determined in Evans' analysis. The authors, therefore, undertook a review of topographic analysis to develop a system to incorporate upstream drainage area. In the course of this review, other changes were made to improve the analysis.

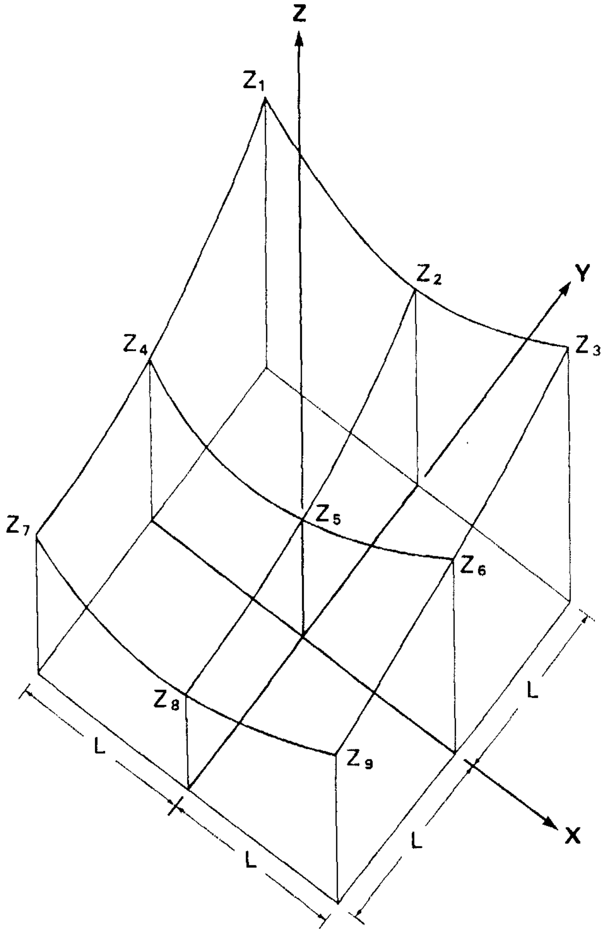

Figure 1. 3 × 3 altitude submatrix

MODIFICATIONS

Evans' quadratic equation, equation 1, need not pass exactly through the nine elevations of the 3 × 3 altitude submatrix. If the surface represented by equation 1 does not coincide with the nine original elevations, it is questionable whether equation 1 represents the land surface accurately. Therefore, the first modification is that of choosing a surface which does pass exactly through the nine submatrix elevations. The appropriate surface is produced by the partial quartic equation:

$$Z = Ax^2y^2 + Bx^2y + Cxy^2 + Dx^2 + Ey^2 + Fxy + Gx + Hy + I \qquad (2)$$

The nine parameters, $A \ldots I$, can be determined from the nine elevations of the 3 × 3 submatrix by Lagrange polynomials. Equation 2 is more general than equation 1 and can be regarded as being more 'flexible'. If the surface indicated by the nine submatrix elevations is of lower order than equation 2 the appropriate coefficients will equal zero and not affect the equation (if the surface is quadratic A, B and C will equal zero and if the surface is a plain $A \ldots F$ will equal zero). It cannot be stated, however, that either equation 1 or 2 will fully replicate the actual land surface between grid points. It is necessary to choose an appropriate grid mesh distance to minimize this problem.

The relationships between the nine parameters of equation 2 and the nine submatrix elevations are:

$$A = [(Z_1 + Z_3 + Z_7 + Z_9)/4 - (Z_2 + Z_4 + Z_6 + Z_8)/2 + Z_5]/L^4 \qquad (3)$$

$$B = [(Z_1 + Z_3 - Z_7 - Z_9)/4 - (Z_2 - Z_8)/2]/L^3 \qquad (4)$$

$$C = [(-Z_1 + Z_3 - Z_7 + Z_9)/4 + (Z_4 - Z_6)/2]/L^3 \qquad (5)$$

$$D = [(Z_4 + Z_6)/2 - Z_5]/L^2 \qquad (6)$$

$$E = [(Z_2 + Z_8)/2 - Z_5]/L^2 \qquad (7)$$

$$F = (-Z_1 + Z_3 + Z_7 - Z_9)/4L^2 \qquad (8)$$

$$G = (-Z_4 + Z_6)/2L \qquad (9)$$

$$H = (Z_2 - Z_8)/2L \qquad (10)$$

$$I = Z_5 \qquad (11)$$

Z_1 to Z_9 are the nine submatrix elevations numbered systematically as shown in Figure 1. Z_5 is the centre point ($x = y = 0$). L is the distance between matrix points in the row and column directions and must be in the same units as Z.

The need for the second modification is dictated by the application of topographic indices to the prediction of processes of flow and sediment transport. Since the estimation of hydraulic and sediment transport parameters (mean flow velocity, average shear stress, stream power) traditionally require slopes in dimensionless terms ($\mathrm{d}H/\mathrm{d}L \approx \sin\theta$) rather than in terms of degrees, it is desirable to measure slope in dimensionless terms. Also, changes in the hydraulic and sediment transport parameters are related to the rate of change in (dimensionless) slope so curvature in terms of 1/LU is superior to curvature computed in terms of degrees/LU. However, the option to retain the other units is available in the program.

Finally, one addition is made to Evans' terrain analysis system, again to facilitate flow and sediment transport prediction. Using the aspect values determined for each point in the watershed, two 'upslope' parameters are determined. These parameters are UPAREA and UPDIST. UPAREA is the drainage area contributing to a point's flow of water and sediment and UPDIST is the longest travel path to an upslope divide. These parameters are determined at every matrix point but would only have meaning when the altitude matrix includes an entire drainage area.

TOPOGRAPHIC INDICES

The topographic indices are found by differentiating equation 2 and solving the resulting equation for the central point of the 3 × 3 submatrix ($x = y = 0$). The slope is the first derivative of Z with respect of S, where S is in the aspect direction (θ).

$$\text{SLOPE} = \partial Z/\partial S = G\cos\theta + H\sin\theta \tag{12}$$

Since, at the origin, $\cos\theta = -G/(G^2+H^2)^{1/2}$ and $\sin\theta = -H/(G^2+H^2)^{1/2}$,

$$\text{SLOPE} = -(G^2+H^2)^{1/2} \tag{13}$$

The negative sign indicates that the direction, θ, is down-slope and is, by convention, ignored. The maximum-slope direction or aspect, θ, is found by differentiating equation 12 to find its minimum.

$$\partial\text{SLOPE}/\partial\theta = -G\sin\theta + H\cos\theta = 0 \tag{14}$$

or

$$\theta = \arctan(-H/-G) \tag{15}$$

The signs of the numerator and denominator of equation 15 determine in which quadrant θ lies. The curvature for any direction, ϕ, is the second derivative of Z with respect to S.

$$\text{Curvature} = \partial^2 Z/\partial S^2 = 2(D\cos^2\phi + E\sin^2\phi + F\cos\phi\sin\phi) \tag{16}$$

The two directions of meaningful curvature are in the direction of the slope ($\phi = \theta$), giving profile curvature, and transverse to the slope ($\phi = \theta + \pi/2$), giving planform curvature.

$$\begin{aligned}\text{PROFC} &= -2(D\cos^2\theta + E\sin^2\theta + F\cos\theta\sin\theta)\\ &= -2(DG^2 + EH^2 + FGH)/(G^2+H^2)\end{aligned} \tag{17}$$

$$\begin{aligned}\text{PLANC} &= 2(D\sin^2\theta + E\cos^2\theta - F\sin\theta\cos\theta)\\ &= 2(DH^2 + EG^2 - FGH)/(G^2+H^2)\end{aligned} \tag{18}$$

It should be noted that equations 16, 17, and 18 do not give true curvature but are, in fact, directional derivatives. The mathematical definition of curvature (in units of radians/LU) is a function of both the second derivative and the first derivative (slope in dimensionless terms) given by:

$$K = (\partial^2 Z/\partial S^2)/\{1 + (\partial Z/\partial S)^2\}^{3/2} \tag{19}$$

Whether equation 16 or 19 is used to determine curvature, the same topographic properties are being measured. The program calculates curvature in either set of units at the user's option. Aspect's units are degrees, slope is dimensionless (LU/LU) and curvature's dimensions are 1/LU. As curvature is generally small, it is desirable to multiply equations 17 and 18 by 100, giving curvature dimensions of 1/100 LU.

The determination of UPAREA for each point is based on the assumption that each point represents one grid square of area (L^2) and routes this area, plus its upstream area, to a neighbouring point. A point receives area from any of its eight neighbouring points which have slopes facing this central point. If no neighbour is pointing towards the central point the central point's upstream area is zero. UPDIST is determined at the same time as UPAREA. A point, however, can only receive drainage distance from one of its eight neighbouring points, the one with the largest drainage distance, plus the distance from the contributing point to the centre point. This distance is either L or $\sqrt{2}\,L$ for a lateral or diagonal contributing point, respectively. The program sweeps repetitively through the altitude matrix in four directions. This makes a specific matrix orientation unnecessary and the failure of the program to determine upslope indices at any point unlikely.

PROGRAM CODING

A program written in FORTRAN 66 uses the previously outlined procedure to determine aspect, slope, profile curvature, plan curvature, drainage area, and upslope distance for points within an altitude matrix. The altitude matrix must be rectangular with unknown altitudes, those altitudes outside the catchment under analysis, given a zero value. The matrix need not have a northern orientation, as the angle the matrix is rotated from north is a required input datum. There are three options available to the user: (1) Slopes can be calculated in either dimensionless terms or degrees; (2) and (3) profile curvature and/or plan curvature can be calculated in units of 1/100 LU or degrees/100 LU.

A subprogram, OUTPUT, produces line printer maps of the six topographic indices plus altitude. Figures 2 to 8 are from an analysis of the Goodwin Creek watershed in Panola County, northwest Mississippi. The 8·2 square mile (21 km^2) watershed was hand digitized at a 200 ft (61 m) grid mesh from a 1:5000 scale, 2 ft (0·61 m) contour interval topographic map produced by the U.S. Army Corps of Engineers. Altitude and slope are mapped showing 10 per cent intervals of the maximum value (Figures 2 and 3). Profile curvature and plan curvature are also mapped at 10 per cent intervals of the (absolute) maximum with a minus sign overstrike for

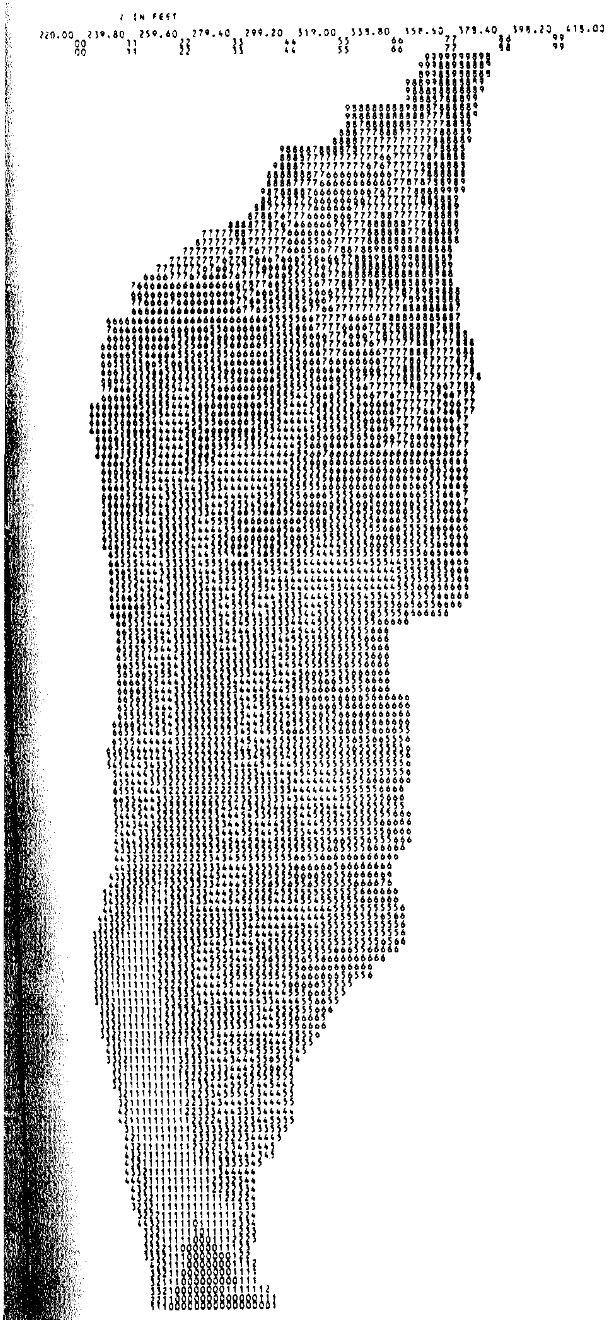

Figure 2. Line printer map of altitude

Figure 3. Line printer map of slope

values less than zero (Figures 4 and 5). Aspect is mapped showing eight directions (Figure 6). A good representation of the drainage network is shown by the map of upstream area (Figure 7), also mapped at 10 per cent intervals. Use of a 10 per cent interval can be inadequate for this mapping because a small drainage network (with respect of the catchment size) often captures most of the upstream area and runoff. It appears that upstream area varies somewhat geometrically with distance so the square-root of upstream area is also

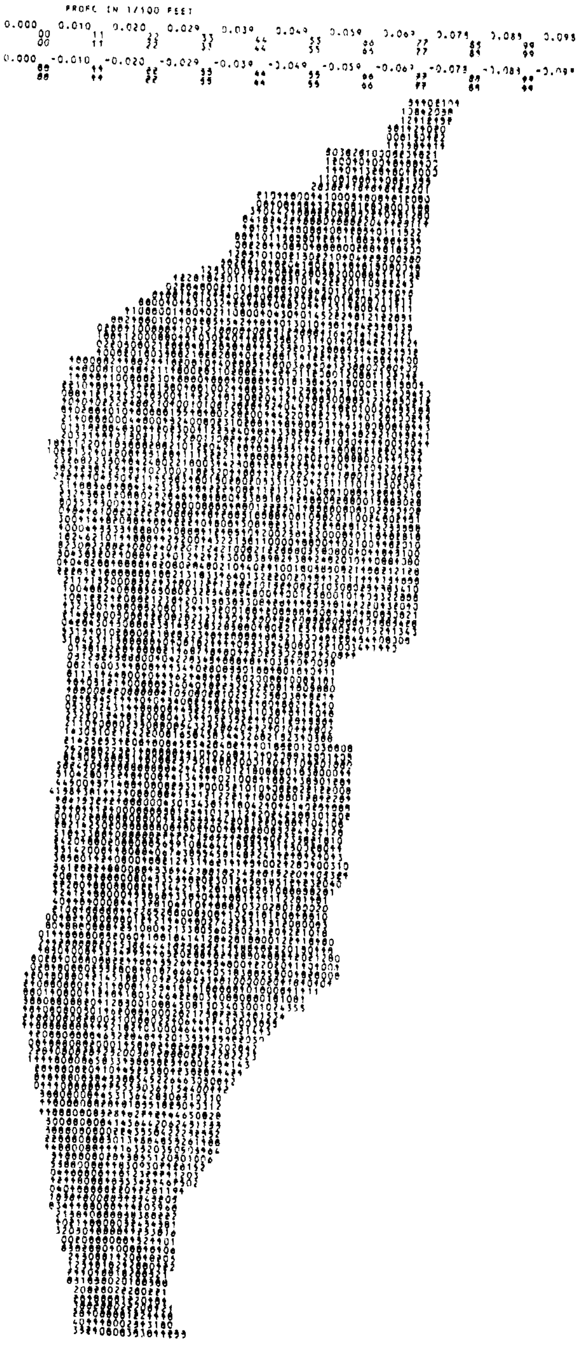

Figure 4. Line printer map of profile curvature

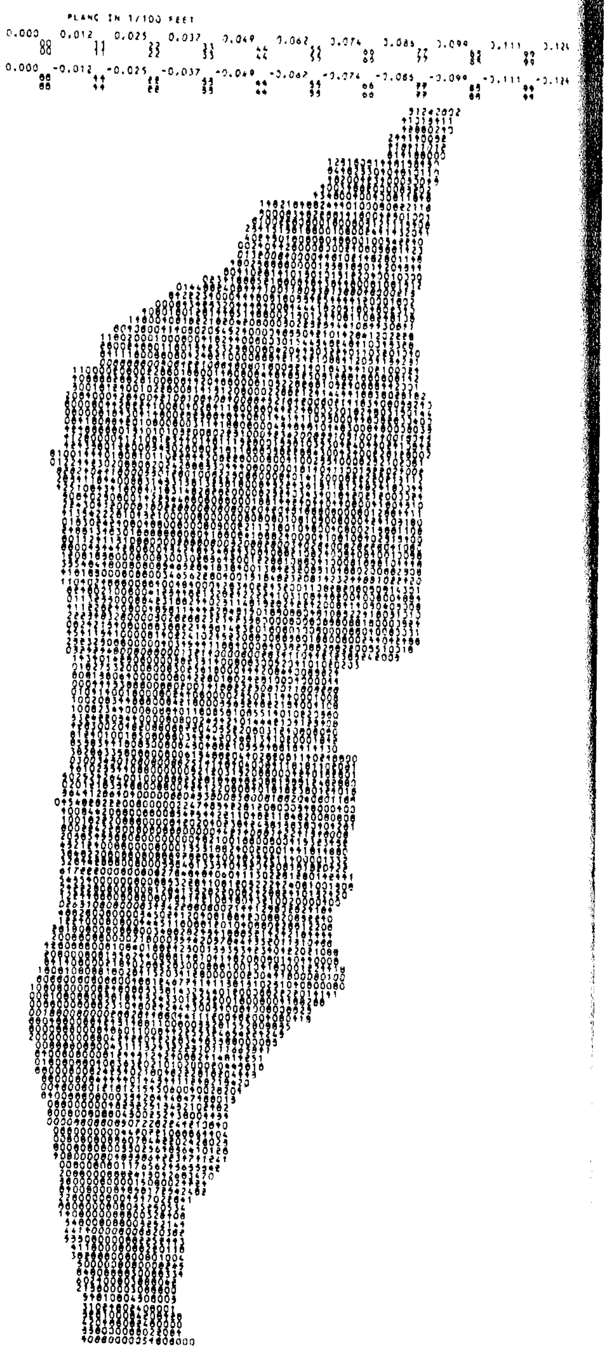

Figure 5. Line printer map of planform curvature

mapped at 10 per cent intervals (Figure 8), which gives a more detailed representation of the drainage network. A similar map (not included) to Figure 8 is also produced for the drainage distance parameter. These line printer maps give quick visualization of the range and distribution of the topographic parameters. Direct quantitative analysis from these maps would be difficult as each symbol represents a range of values (i.e. each symbol in Figure 3 represents slopes over a range of about 1·3 per cent).

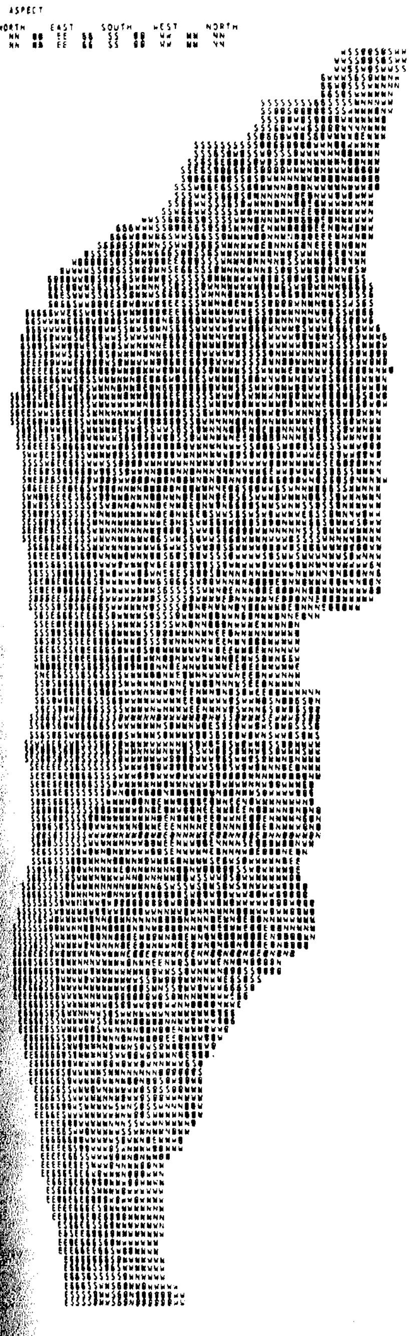

Figure 6. Line printer map of aspect

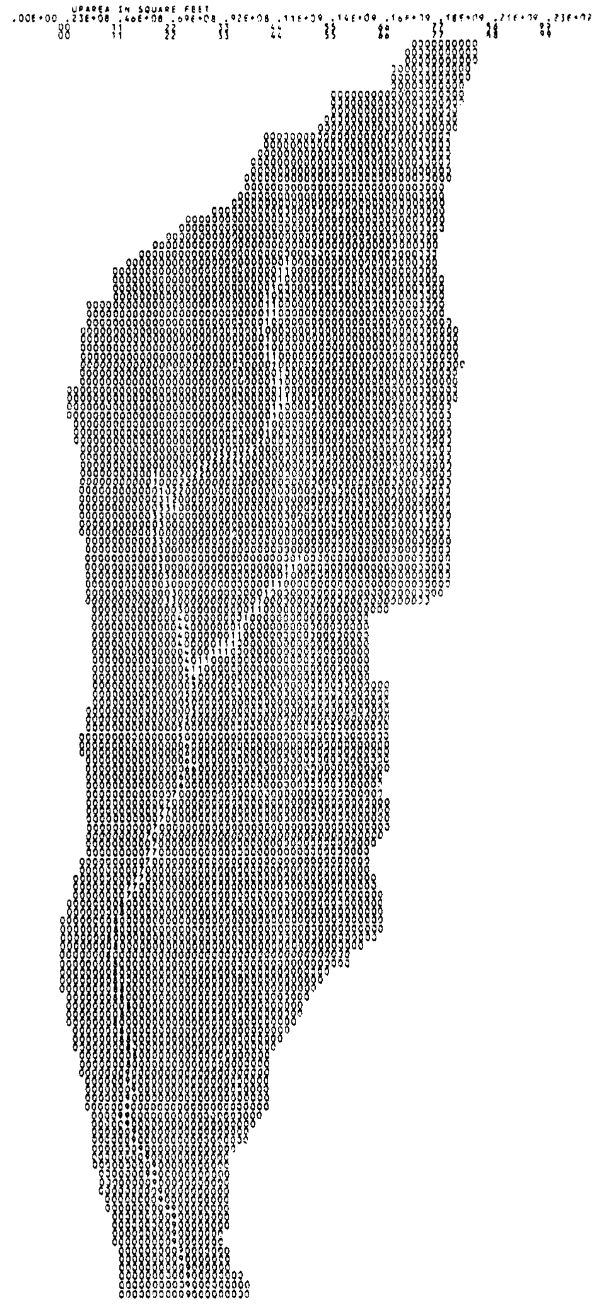

Figure 7. Line printer map of upstream area

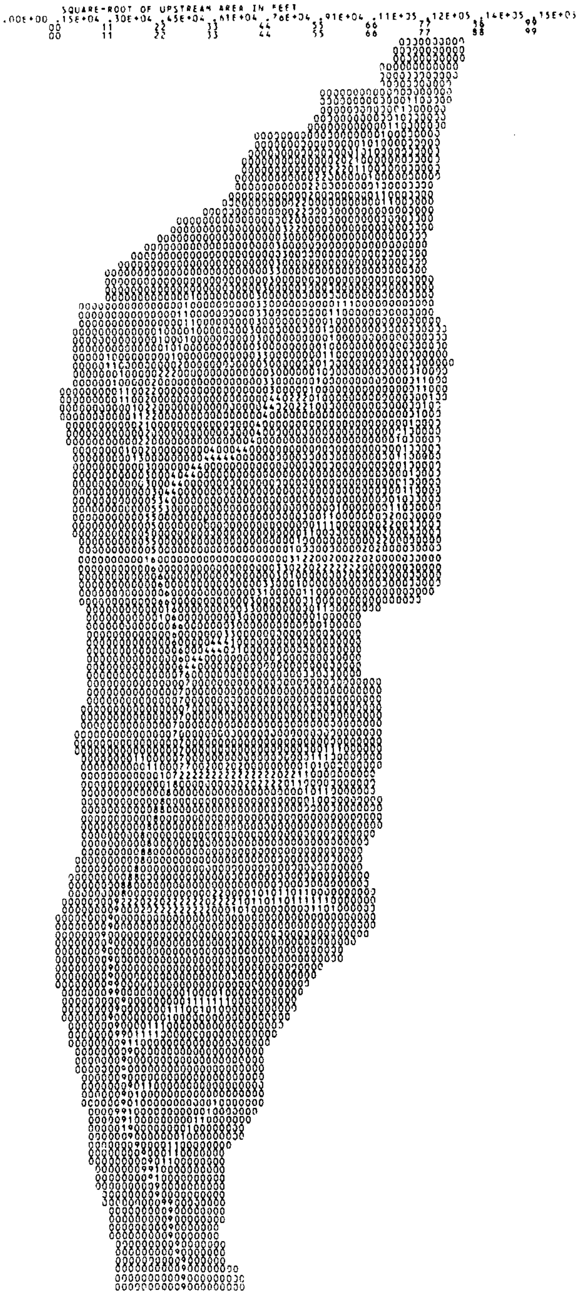

Figure 8. Line printer map of the square-root of upstream area

DISCUSSION

The use of a partial quartic or quadratic equation for land surface analysis assumes that the surface represented by the altitude matrix has a continuous first derivative. Although this is a reasonable assumption for most land surfaces there are instances when this assumption can cause problems. When a land surface contains an abrupt break in slope, such as a river bank, special treatment must be made for grid points which fall within such areas.

The elevations of grid points which fall between river banks should be adjusted to a height slightly lower than the top of bank elevation. This avoids the occurrence of extremely high values of slope and curvature and spurious values of aspect. This is especially a problem if the grid mesh distance is greater than the river width. When this happens, only occasional grid points fall between the river banks causing an apparent hole in the land surface. If this occurs, it is likely that the drainage area and drainage distance routine will fail.

Choice of an appropriate grid mesh distance is clearly an important consideration. Evans (1979) states that analyses of two land surfaces cannot be directly compared if the grid mesh or the elevation precision differ. Also, choice of an appropriate grid mesh is dependent on the variation of slope and aspect (profile and plan curvature) and on the precision of elevation data. Too large a grid mesh will miss the topographic features of a land surface while too small a grid mesh will require excessive, precise data giving only minor land features.

APPLICATIONS

As shown by Evans (1980), there are many, widely varied uses for topographic analysis. Aspect and slope were used by Foyster (1973) in a grid square technique for mapping evapotranspiration. Anderson and Burt (1978) and O'Loughlin (1981) showed that the presence of hollows and spurs significantly affects rates of hillslope discharge. Burt and Butcher (1985) related a variety of topographic indices to soil water potential, saturation depth, and slope discharge. Thorne and Zevenbergen (1986) used a combination of drainage area, slope and planform curvature to predict areas of initial gully incision in croplands. Although upstream area appears as a variable and is found to be significant in Burt and Butcher's (1985) analysis, their method for obtaining this topographic index was not automated, as it is in the authors' analysis.

CONCLUSION

The topographic analysis presented here represents an initial analysis of a land surface, with line printer maps provided as convenient visual output. Any additional analyses, whether statistical, hydrologic, or geomorphic, can be accomplished by subprograms written by the user. The program is written in standard FORTRAN 66 and has been run on an ICL 2900 series computer using both the FORTRAN 66 and 77 compilers. For compatible systems, a copy of the program can be obtained by sending a magnetic tape to either author at the U.K. address.

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REFERENCES

Anderson, M. G. and Burt, T. P. 1978. 'The role of topography in controlling throughflow generation', *Earth Surface Processes*, 3(4), 331-334.

Burt, T. P. and Butcher, D. P. 1985. 'Topographic controls of Soil Moisture Distributions', *Journal of Soil Science*, **36**, 469-486.

Evans, I. S. 1972. 'General geomorphometry, derivatives of altitude and descriptive statistics', in Chorley, R. J. (Ed.), *Spatial Analysis in Geomorphology*, p. 36, Methuen, London.

Evans, I. S. 1979. 'An integrated system of terrain analysis and slope mapping', Final report on Contract DA-ERO-591-73-G0040, U.S. Army European Research Office, London.

Evans, I. S. 1980. 'An integrated system of terrain analysis and slope mapping', *Zeitschrift für Geomorphologie, N.F., Supplementband*, **36**, 274-295.

Foyster, A. M. 1973. 'Application of the grid square technique to mapping of evapotranspiration', *Journal of Hydrology*, **19**, 205-226.

O'Loughlin, E. M. 1981. 'Saturation regions in catchments and their relationships to soil and topographic properties', *Journal of Hydrology*, **53**, 229-246.

Rhind, D. W. 1972. 'One-sided constraints in automated contouring of drift thicknesses', *Geological Society of America Bulletin*, **83**, 2525–2534.

Sharpnack, D. A. and Akin, G. 1969. 'An algorithm for computing slope and aspect from elevation', *Photogrametric Engineering*, **35**, 247–248.

Thorne, C. R. and Zevenbergen, L. W. 1986. 'Ephemeral gullies as sources of sediment', *Proceedings of the Fourth Federal Interagency Sedimentation Conference*, U.S. Govt. Printing Office, 3-152–3-161.

Tobler, W. R. 1969. 'An analysis of a digitalized surface', in Davis, C. M. (Ed.), *A Study of Land Type*, Univ. of Michigan Dept. of Geography, 59–76.

Young, M. 1978. 'Statistical characterization of altitude matrices by computer', The fifth progress report on Contract DA–ERO–591–73–G0040, U.S. Army European Research Office, London.