

This article was downloaded by:[Qin, C.]
[Qin, C.]

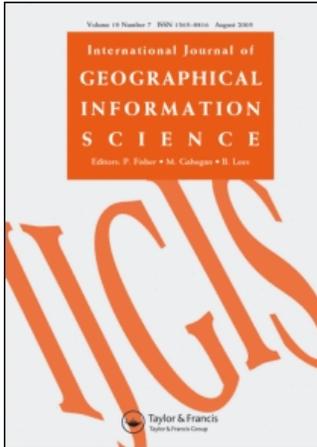
On: 29 March 2007

Access Details: [subscription number 776019716]

Publisher: Taylor & Francis

Informa Ltd Registered in England and Wales Registered Number: 1072954

Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



International Journal of Geographical Information Science

Publication details, including instructions for authors and subscription information:
<http://www.informaworld.com/smpp/title-content=t713599799>

An adaptive approach to selecting a flow-partition exponent for a multiple-flow-direction algorithm

To cite this Article: , 'An adaptive approach to selecting a flow-partition exponent for a multiple-flow-direction algorithm', International Journal of Geographical Information Science, 21:4, 443 - 458

xxxx:journal To link to this article: DOI: 10.1080/13658810601073240

URL: <http://dx.doi.org/10.1080/13658810601073240>

Full terms and conditions of use: <http://www.informaworld.com/terms-and-conditions-of-access.pdf>

This article maybe used for research, teaching and private study purposes. Any substantial or systematic reproduction, re-distribution, re-selling, loan or sub-licensing, systematic supply or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

© Taylor and Francis 2007

Research Article

An adaptive approach to selecting a flow-partition exponent for a multiple-flow-direction algorithm

C. QIN*†, A.-X. ZHU‡, T. PEI†, B. LI†, C. ZHOU† and L. YANG†

†State Key Laboratory of Resources and Environmental Information System, Institute of Geographical Sciences and Natural Resources Research, CAS, Beijing 100101, China

‡Department of Geography, University of Wisconsin Madison, 550N, Park Street, Madison, WI 53706-1491, USA

(Received 2 February 2006; in final form 18 October 2006)

Most multiple-flow-direction algorithms (MFDs) use a flow-partition coefficient (exponent) to determine the fractions draining to all downslope neighbours. The commonly used MFD often employs a fixed exponent over an entire watershed. The fixed coefficient strategy cannot effectively model the impact of local terrain conditions on the dispersion of local flow. This paper addresses this problem based on the idea that dispersion of local flow varies over space due to the spatial variation of local terrain conditions. Thus, the flow-partition exponent of an MFD should also vary over space. We present an adaptive approach for determining the flow-partition exponent based on local topographic attribute which controls local flow partitioning. In our approach, the influence of local terrain on flow partition is modelled by a flow-partition function which is based on local maximum downslope gradient (we refer to this approach as MFD based on maximum downslope gradient, MFD-md for short). With this new approach, a steep terrain which induces a convergent flow condition can be modelled using a large value for the flow-partition exponent. Similarly, a gentle terrain can be modelled using a small value for the flow-partition exponent. MFD-md is quantitatively evaluated using four types of mathematical surfaces and their theoretical 'true' value of Specific Catchment Area (SCA). The Root Mean Square Error (RMSE) shows that the error of SCA computed by MFD-md is lower than that of SCA computed by the widely used SFD and MFD algorithms. Application of the new approach using a real DEM of a watershed in Northeast China shows that the flow accumulation computed by MFD-md is better adapted to terrain conditions based on visual judgement.

Keywords: Digital elevation model (DEM); Multiple flow direction algorithm (MFD); Terrain analysis; Flow accumulation; Geographic information system (GIS)

1. Introduction

Flow direction is one key factor in many research fields (such as distributed hydrological modelling, watershed feature extraction, geomorphology, soil erosion, etc.) (Beven and Kirkby 1979, Moore *et al.* 1991, Wilson and Gallant 2000) and thus becomes an important subject of terrain analysis. The computation of flow direction is based on the assumption that the local hydraulic gradient can be estimated with

*Corresponding author. Email: qincz@lreis.ac.cn

the local slope gradient which is derived from the DEM, especially the gridded DEM (Beven and Kirkby 1979, O'Loughlin 1986, Wilson and Gallant 2000). Therefore, most flow-direction algorithms use gridded DEM to determine the flow direction of a point according to the elevations in a 3×3 window around it (Wilson and Gallant 2000).

Current flow-direction algorithms can be classified into two main types according to the flow-direction schemes used: single flow direction (SFD) and multiple flow direction (MFD) algorithms (Wolock and McCabe 1995). The basic idea of SFD algorithms is that all water from a point (or pixel) should flow into one and only one neighbouring pixel which has the lowest elevation. This idea of the steepest descent direction is easy to implement, and suited for modelling convergent flow. Many SFD algorithms have been proposed and frequently used in many flow routing applications (O'Callaghan and Mark 1984, Martz and de Jong 1986, Jenson and Domingue 1988, Fairfield and Leymarie 1991, Lea 1992). However, SFD has evident defects, i.e. the inability to model divergence of flow and the production of parallel flow (Freeman 1991, Fairfield and Leymarie 1991, Wilson and Gallant 2000). A detailed discussion of SFD and its shortcomings are beyond the scope of this paper, and interested readers are referred to Bertolo (2000) for more discussion on SFD.

MFD assumes that flow from the current position could drain into more than one downslope neighbouring pixel. The flow-partitioning proportion among downslope neighbouring pixels is determined based on slope gradient (Quinn *et al.* 1991). Much research has shown that MFD is obviously better than SFD when the spatial pattern of hydrological parameters (such as flow accumulation and topographic index) is computed (Moore *et al.* 1993a, Wolock and McCabe 1995, Pan *et al.* 2004). The key issue in MFD is how to partition the flow into multiple downslope pixels. A commonly used approach for modelling divergent flow is the fixed-exponent strategy (Freeman 1991, Quinn *et al.* 1991), although other approaches have been examined (Costa-Cabral and Burges 1994, Tarboton 1997). The fixed-exponent strategy is unsuitable when the terrain conditions include both convergence and divergence because the exponent cannot be altered in response to local terrain conditions during the execution of the programme. In other words, this approach is not adaptive to local terrain conditions which are important for partitioning local flow. Some efforts have been made to model the flow partition in complex terrain using a varying exponent strategy (e.g. Quinn *et al.* 1995, Kim and Lee 2004). However, the effect of local terrain conditions on the dispersion of local flow has still not been effectively modelled in these efforts. The shortcomings of current MFDs are further discussed in the next section.

This paper presents an adaptive approach for varying a flow-partition exponent based on local terrain conditions. Section 2 reviews current MFD algorithms and highlights the key problems associated with the fixed exponent strategy. Section 3 presents the new flow-partition approach. Section 4 provides a quantitative evaluation of the new approach using mathematical surfaces. Section 5 presents an application of the method using a real watershed in Northeast China. Conclusions are made in section 6.

2. MFD algorithms based on a flow-partition exponent

In MFD, the cell with the steeper descent, among all downslope neighbouring cells, will receive more flow. Quinn *et al.* (1991) made the original contributions for MFD

neighbouring cell with the steepest downslope, which should be similar to the situation modelled by SFD. The flatter the terrain condition, the more divergent the flow. Thus, the flow from the centre pixel should be distributed more evenly to the neighbouring downslope pixels. Under MFD-Quinn which sets $p=1$ (Quinn *et al.* 1991), the flow is partitioned in a uniform manner for all three areas, and the partition of flow cannot adapt to different local terrain conditions, no matter how steep or flat (figure 1). This deficiency is also seen in all other MFD algorithms which set p as a constant (fixed p). Although more convergent flow might be modelled using a higher p value (Holmgren 1994), for an MFD with a fixed p value, the p value cannot be altered according to local terrain conditions during the execution of the algorithm. Thus, the particular fixed p value might be good for some locations but not for others. This means that MFDs with a fixed p value cannot adapt to local terrain conditions.

Some algorithms try to set up a flow-partition scheme to accomplish both MFD for a divergent surface and SFD for a convergent surface. Moore *et al.* (1993b) set up an FD8/FRho8 method. FD8/FRho8 requires the specification of a critical upslope contributing area for channel initiation. Then, the MFD with a fixed p value proposed by Freeman (1991) is applied in upland areas above defined channels, and the D8 (O'Callaghan and Mark 1984) or Rho8 (Fairfield and Leymarie 1991) algorithm is applied below the points of channel initiation. But the appropriate value of a critical upslope contributing area might vary depending on the characteristics of catchment (Güntner *et al.* 2004). The transition from MFD to SFD can cause an irregularity in the frequency distribution of the contributing area (Wilson and Gallant 2000).

Pilesjö *et al.* (1998) designed a 'form-based' MFD. The basic idea is that the topographic form of a sub surface (i.e. 'form') dictates the flow distribution. If the form is concave or flat, SFD is applied. If the form is convex, the MFD-Quinn algorithm is applied. This is a clever idea, but the method is also very sensitive to the quality of DEM because small errors in DEM could shift the form from the convex to the concave or vice versa. However, flow on convex forms is still modelled with a fixed flow-partition exponent.

Quinn *et al.* (1995) described a MFD algorithm where the flow-partition exponent was changed continuously from 1 (full dispersion) to a large value (SFD) as the contributing area increased, giving a smooth change from MFD to SFD. They used the MFD-Quinn algorithm to compute a spatial distribution of flow accumulation. Then, an 'optimal' channel initiation threshold (CIT) was identified for computing the flow-partition exponent using the following equation:

$$p = (A/\text{thresh} + 1)^h \quad (2)$$

where A is the pre-computed accumulation of the interested cell; thresh is the accumulation threshold of the application region, the same as the channel initiation of Moore *et al.* (1993b); and h is a positive constant. The larger the value of h , the faster the method approaches SFD. Some researchers also focused on how to acquire a varying p value according to the accumulation distribution pre-computed by other SFD or MFD. An example is the SDFAA method which computes a p value by a genetic algorithm (Kim and Lee 2004). These MFD algorithms are based on an assumption that the pre-computed accumulation distribution can sufficiently reflect the effects of local terrain conditions on flow distribution. But flow accumulation is not actually a 'local' topographic attribute. The accumulation of a

cell is affected by all cells in its upland areas. Moreover, different flow-direction algorithms might produce very different accumulation distributions. So the rationale of this assumption is questionable, and this type of MFD is not independent of other flow-direction algorithms. Furthermore, the identification of 'optimum' CIT is subjective and complicated (Quinn *et al.* 1995, Güntner *et al.* 2004). Despite these deficiencies, the varying flow-partition exponent could still yield more reasonable results than the fixed exponent (Quinn *et al.* 1995, Kim and Lee 2004).

The above discussion leads us to see both the deficiencies of current MFD algorithms and the need for an adaptive strategy for determining the flow-partition exponent according to local terrain conditions. In the next section, we present an adaptive approach to varying the exponent p , which is based on local topographic attribute that reflects the effects of local terrain conditions on flow distribution.

3. Adaptive approach to varying the flow-partition exponent for MFD

3.1 Basic idea

We believe that the local terrain conditions control flow partitioning at each cell. It is more reasonable to use local topographic conditions to determine the flow-partition exponent than to use global topographic attributes (such as flow accumulation). Using local topographic conditions also makes the algorithm more adaptive to local terrain conditions which directly control the local flow partition. This idea results in the development of a new flow-partition approach to MFD. There are two steps in building this new approach. The first is the selection of the local topographic attribute (e) which can directly describe the effect of local terrain on flow partitioning. The second step is the construction of a function of e ($f(e)$) for computing the flow-partition exponent which is adaptive to the local terrain conditions. So, the varying flow-partition exponent is determined through a flow-partition exponent function $f(e)$ based on local terrain conditions. The two steps are discussed below in detail.

3.2 Selection of local topographic attribute

The candidates of local topographic attribute for our flow-partition scheme should be those which not only reflect the local terrain conditions but also control the local drainage. It is natural to use something similar to a local slope gradient because the local hydraulic gradient is estimated with the local slope gradient, which is computed by the centre cell and all of its neighbouring cells, according to the basic assumption of flow-direction algorithms (Beven and Kirkby 1979). Hjerdt *et al.* (2004) argued that the downslope topography is an important factor for estimating a hydraulic gradient. Among the topographic attributes related to the local slope gradient, Güntner *et al.* (2004) also showed that the local downslope gradient computed by the centre cell and its downslope neighbouring cell can quantify flow of water better than the local slope in explaining the soil water content of catchment. Therefore, we use downslope gradient as the local topographic attribute to model the impact of local terrain on flow partition.

We consider three downslope gradients as candidates for our MFD scheme: the maximum downslope gradient, the minimum downslope gradient, and the average downslope gradient. We determine which candidate of these local topographic attributes is the most appropriate for the new flow-partition scheme based on two

principles. The first is that the chosen local topographic attribute can express the change in local terrain condition which has evident geomorphological meanings for the change in flow partitioning. As discussed before, the flow-partition scheme should be more similar to SFD when the terrain condition consists of a higher downslope gradient in the steepest descent direction. Most directly, the maximum downslope gradient, among the three candidates, can quantify the steepness in the steepest descent direction.

The other principle for choosing the local topographic attribute is related to the sensitivity to subtle variations in DEM. The local topographic attribute should not be too sensitive to subtle variations of DEM because not only is the effect of subtle DEM variations on flow partition often small, but also the likelihood of subtle DEM variations being errors is high. Theoretically speaking, the subtle variations of DEM have a much more marked effect on the minimum downslope gradient than the maximum downslope gradient. The average downslope gradient is also sensitive to the subtle variations in DEM error because these subtle variations can change the number of neighbouring downslope cells. The maximum downslope gradient should be less sensitive to the DEM error than the minimum or average downslope gradients.

Figure 2 illustrates the sensitivity of these three candidates and slope gradients to subtle variations in DEM for the watershed described in section 5. Through visual comparison, all of them show the variation of local terrain conditions, more or less. But there are many disorderly and unsystematic breaks in the image of an average downslope gradient and a minimum downslope gradient. The spatial distribution of the maximum downslope gradient, which is similar to that of a slope gradient, follows the terrain conditions on the contour maps (see also figure 4) the best. Therefore, the maximum downslope gradient meets the two principles best and was chosen as the local topographic attribute for the new flow-partition scheme.

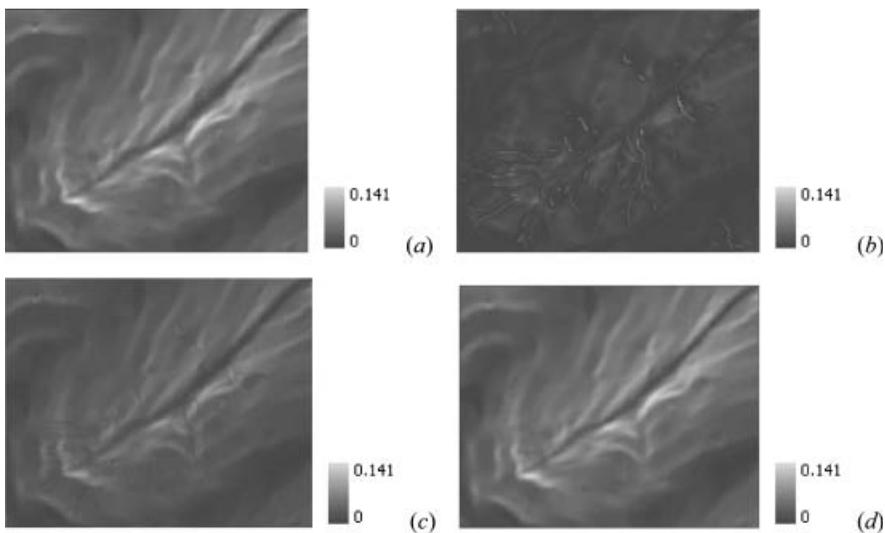


Figure 2. Comparison of (a) maximum downslope gradient, (b) minimum downslope gradient, (c) average downslope gradient, and (d) slope gradient.

3.3 Determination of flow-partition exponent using the maximum downslope gradient

The construction of the function $f(e)$ for determining the flow-partition exponent using the maximum downslope gradient includes two issues: the function form of $f(e)$ and its upper and lower bounds. The principle used in constructing $f(e)$ is that $f(e)$ can properly model the effects of the maximum downslope gradient on flow partitioning. Furthermore, the function form should be as simple as possible. When the maximum downslope gradient is large, the value of $f(e)$ should be large accordingly to model the convergent flow. If the maximum downslope gradient is small, the corresponding $f(e)$ should also be small to model the divergence of flow. A linear function would meet both conditions. Therefore, we chose a linear function of the maximum downslope gradient as $f(e)$. Equation (3) gives the form of this linear function

$$f(e) = \begin{cases} p_l & ; (e \leq e_{\min}) \\ \frac{e - e_{\min}}{e_{\max} - e_{\min}} \times (p_u - p_l) + p_l & ; (e_{\min} < e < e_{\max}) \\ p_u & ; (e \geq e_{\max}) \end{cases} \quad (3)$$

where e is the tangent value of the maximum downslope gradient ($\tan \beta$ and β being the maximum downslope gradient); $f(e)$ is the flow partition function; p_u and p_l are the upper and lower bounds of $f(e)$ and are used as the p values representing completely divergent and convergent flows, respectively (see discussion below); and e_{\min} and e_{\max} are the e values which are associated with p_l and p_u , respectively. The determination of the bounds (i.e. p_u and p_l) for $f(e)$ depends on many researchers' comparison and discussion about the best value of the flow-partition exponent for modelling the complete divergence or convergence of flow. Among integers, $p=1$ is proposed for the complete divergence of flow (Quinn *et al.* 1991, Holmgren 1994, Pilesjö and Zhou 1997). Freeman (1991) proposed $p=1.1$ as the best value after he analysed the instances of p being 1, 1.1, and 1.25, respectively. So, we choose $p=1.1$ as the lower bound for $f(e)$. Both Holmgren (1994) and Quinn *et al.* (1995) recommended $p=10$ for modelling the single directional flow. Their recommendation is taken here as the upper bound of $f(e)$. So, the domain of $f(e)$ is defined as [1.1, 10].

We set $e_{\min}=0$ and $e_{\max}=1$ ($\tan 45^\circ$) for this experiment. This means that the flow-partition exponent is set to p_u to model the single flow direction when the local maximum downslope gradient is 45° or above, and to p_l when the local maximum downslope gradient is 0. It is understandable to choose 0° for e_{\min} because we expect flat areas to be typical areas of divergent flow. The determination of 1 (or $\tan 45^\circ$) to be associated with p_u is made under an assumption that a location with a maximum downslope gradient of 45° or above will have a complete convergent flow. In addition, the 45° for e_{\max} allows us to capture most of the gradient values for flow partition because slope gradient values for most terrains are in the range of $0-45^\circ$, and multiple flow directions often occur in areas with low gradient values ($<45^\circ$). Although the determination of $e_{\min}=0$ and $e_{\max}=1$ are slightly arbitrary, these settings are made to illustrate the advantages of this adaptive approach to varying flow-partition exponent for MFD algorithms. A detailed study on the effect of these settings on the algorithm will be reported in a separate paper.

With the above settings, the final form of the flow-partition exponent function is as follows:

$$f(e) = 8.9 \times \min(e, 1) + 1.1 \quad (4)$$

where $\min(e, 1)$ is the minimum between e and 1.

The new MFD algorithm proposed in this paper then takes the following form:

$$d_i = \frac{(\tan \beta_i)^{f(e)} \times L_i}{\sum_{j=1}^8 (\tan \beta_j)^{f(e)} \times L_j} \tag{5}$$

where $f(e)$ is computed from equation (4), and the definition of other symbols is the same as that in equation (1). We name the new MFD algorithm MFD-md (MFD algorithm based on the maximum downslope gradient).

Figure 3 shows flow partitioning by MFD-md for the same three areas shown in figure 1. The spatial grid resolution is 1 m. When the terrain is steep, MFD-md drains the flow mainly into the neighbouring cell in the direction with the steepest downslope, which is similar to the situation modelled by SFD. When the terrain becomes flatter, MFD-md partitions the flow adaptively and more evenly distributes the flow among all downslope neighbouring cells, which is similar to the result by MFD-Quinn (figure 1). Thus, in virtue of the adaptive approach to varying flow-partition exponents, MFD-md can more accurately model flow partitioning under different terrain conditions.

4. Quantitative assessment for MFD-md

This method for assessing flow-direction algorithms is necessary because different algorithms can produce very different results, even for the same DEM (Wilson and Gallant 2000). The most commonly used method of assessing error is to apply the algorithm to a real DEM. However, this assessment is data-dependent and hard to quantify (Zhou and Liu 2002). A few artificial DEMs were used to show the rationality of the flow-direction algorithm. An example is an artificial cone surface used in Freeman (1991). However, the flow accumulation computed by the MFD algorithm of Freeman (1991) was subjectively judged to be ‘reasonable’ or ‘unreasonable’. If the ‘true’ value of flow accumulation or Specific Catchment

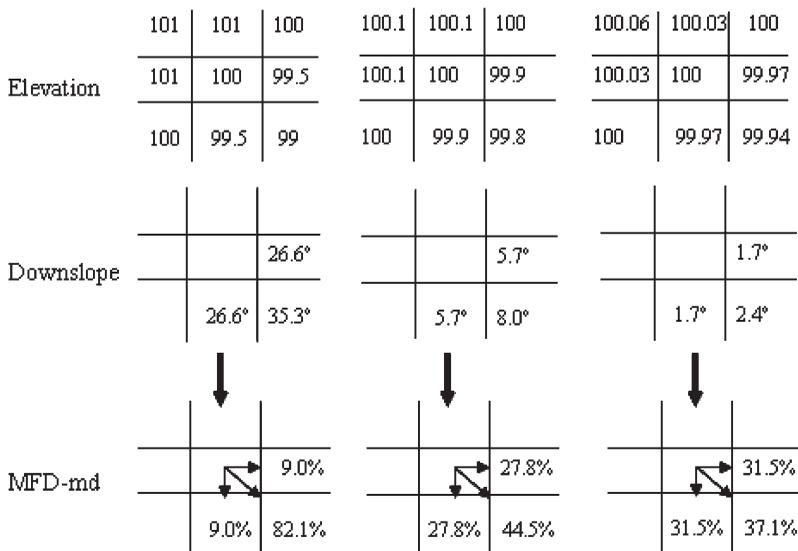
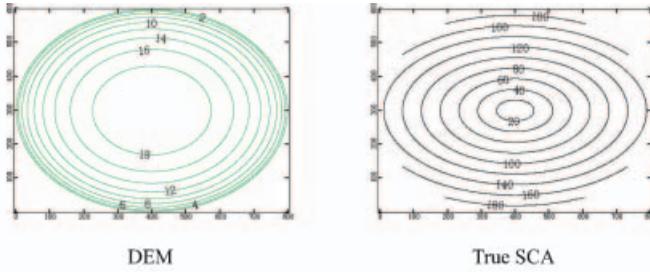
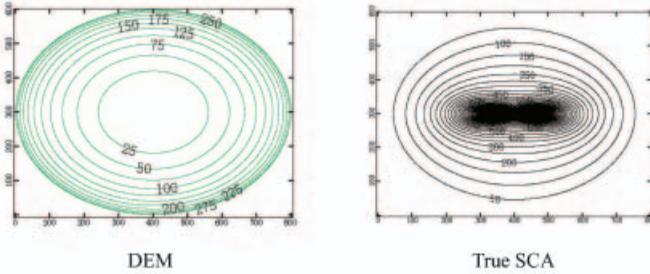


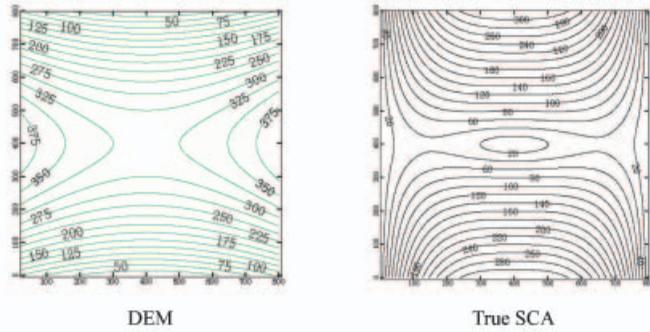
Figure 3. Examples of the MFD-md algorithm.



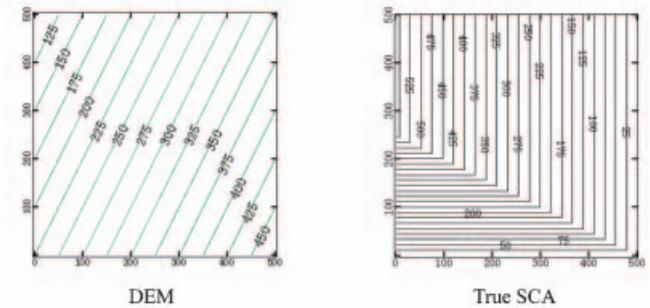
(a) ellipsoid: $\frac{(x-400)^2}{400^2} + \frac{(y-300)^2}{300^2} + \frac{z^2}{20^2} = 1 \quad (z > 0); \quad 0 < x < 800; 0 < y < 600$



(b) inverse ellipsoid: $\frac{(x-400)^2}{400^2} + \frac{(y-300)^2}{300^2} + \frac{(z-300)^2}{300^2} = 1 \quad (z < 300); \quad 0 < x < 800; 0 < y < 600$



(c) saddle: $\frac{(x-400)^2}{2^2} - \frac{(y-400)^2}{1^2} = \frac{z}{0.002}; \quad 0 < x, y < 800$



(d) plane: $z = 0.5x + 0.25y + 100; \quad 0 < x, y < 500; \text{ Slope} = 27.9^\circ$

Figure 4. Mathematical models, the contour maps of artificial surfaces, and the ‘true’ SCA distributions of four typical mathematical surfaces: (a) ellipsoid; (b) inverse ellipsoid; (c) saddle; and (d) plane. The unit represented is in metres.

Area (SCA for short, which is defined as ‘upslope area per unit width of contour’ (Wilson and Gallant 2000)) of artificial surfaces can be pre-determined by mathematical inference, the flow-direction algorithms can then be quantitatively assessed.

There are two quantitative methods with which artificial DEMs are used to assess the error of grid-based flow-routing algorithms. One was developed by Zhou and Liu (2002) and the other by Pan *et al.* (2004). Both create artificial DEMs to simulate typical terrain conditions, such as planar, convergent, divergent, and so on. The root mean square error (RMSE) can be used to assess the errors between the theoretical flow accumulation (or SCA) and the result computed by flow-direction algorithms. Comparatively, the artificial DEMs created by Zhou and Liu (2002) include one more terrain type, i.e. the saddle surface representing the ridge. The ellipsoid surface in Zhou and Liu (2002) is more similar to the practical convex slope than to the cone surface in Pan *et al.* (2004). We use the method developed by Zhou and Liu (2002) to assess MFD-md.

4.1 Method and material

According to Zhou and Liu (2002)’s method, four types of artificial surfaces are constructed using mathematical models. They are based on an ellipsoid (representing convex slopes), an inverse ellipsoid (representing concave slopes), a saddle (representing ridges), and a plane (representing straight slopes) respectively. Figure 4 shows an example of the mathematical models, the contour maps of the artificial surfaces, and the ‘true’ SCA inferred from the mathematical models. The artificial DEMs are generated from the mathematical surfaces at a resolution of 1 m.

Zhou and Liu (2002) take the SCA defined as the upstream length at the point where the contour length tends to zero. The details of computing SCA at any given point on a given mathematical surface are not included in this paper. Interested readers are referred to Zhou and Liu (2002). The error at each cell caused by the flow-direction algorithm can be computed by (Zhou and Liu 2002):

$$E_i = \frac{SCA_i^{\text{true}} - SCA_i}{\text{cell size}} \quad (6)$$

where E_i is the error at the i th cell; SCA_i^{true} and SCA_i are the ‘true’ value of and computed SCA for the cell, respectively. Thus, the RMSE can be computed for the assessment and comparison of the flow-direction algorithms:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^n (E_i)^2}{n}} \quad (7)$$

where n is the number of cells for assessment.

We use the artificial DEMs in figure 4 and the method of Zhou and Liu (2002) to quantitatively assess MFD-md. For comparing MFD-md with classical SFD and MFD, we compute SCAs of artificial DEMs using D8 (a representative SFD) and MFD-Quinn (a representative MFD), and MFD-md, respectively. Here, we do not choose a current MFD with varying flow-partition exponent (e.g. Quinn *et al.* (1995)’s MFD) as the reference MFD. This is mainly because of their dependence on other flow-direction algorithms, as discussed in section 2.

Table 1. RMSE for four mathematical surfaces in figure4 using D8, MFD-Quinn, and MFD-md algorithms.

	Ellipsoid	Inverse ellipsoid	Saddle	Plane
D8	65.76	1906.44	125.05	96.02
MFD-Quinn	6.60	500.51	109.78	412.45
MFD-md	6.09	431.61	97.01	39.77

4.2 Results of quantitative assessment for MFD-md

Table1 lists the RMSE of errors computed by D8, MFD-Quinn, and MFD-md. Under all tested terrain conditions, MFD-md produces the lowest error among the three tested algorithms. The lowest error of all is produced by the ellipsoid surface as opposed to other terrain conditions. This shows that the flow-direction algorithm is more suitable for convex slopes than other typical terrains. All three algorithms obtain their highest errors for inverse surfaces. This may partly be related to the fact that the surface of an artificial inverse ellipsoid has no outlet for flow. For the plane surface (representing the straight slope), MFD-Quinn yields the highest error because MFD-Quinn does not accurately model the flow on the steeper terrain. In summary, MFD-md can model the four kinds of terrain conditions better than D8 and MFD-Quinn.

5. Application of MFD-md

5.1 Data and the pre-processor for depressions and flat areas in DEM

A typical small catchment in the Nenjiang watershed in Northeast China is applied with MFD-md. The catchment is approximately 1.6 km × 1.3 km in size and has a relatively low relief as a whole (figure 5). The DEM is in a regular 10 m grid with more than 20 000 pixels.

A pre-processor for DEM is nearly always necessary for applying the flow-routing algorithms. This is because the real DEM often contains depressions or flat areas which interfere with the execution of flow-routing algorithms (Jenson and Domingue 1988, Kong and Rui 2003). The usual method for dealing with depressions is to fill them (O’Callaghan and Mark 1984). There are two ways for

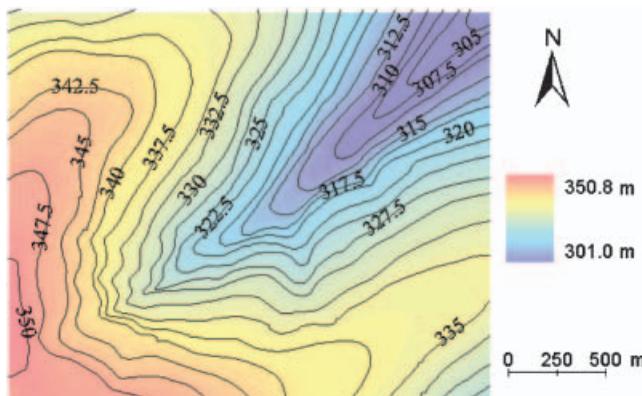


Figure 5. DEM of application area.

processing the flat areas. The first way is to keep the elevation unchanged and assign a flow direction for every cell in flat areas according to a set of rules (Jenson and Domingue 1988). This method is aimed at SFD, not MFD. The second way is to mildly amend the elevation of the cells in flat areas according to the terrain conditions. A good example is the DEM-processing method proposed by Martz and Garbrecht (1998). Their method makes every cell in flat areas have reasonable downslope neighbouring cell(s) without changing the veracity of the DEM. We use Martz and Garbrecht's DEM-processing method to fill depressions and amend elevation of cells in flat areas before MFD-md is applied. The details of this method are not included in this paper, and interested readers are referred to Martz and Garbrecht (1998) for details.

5.2 Application and discussion of MFD-md, SFD, and MFD

The MFD-md and two comparing algorithms are applied after the DEM is pre-processed. The resulting accumulations are shown in figure 6. In the flow accumulation computed by D8 (figure 6(a)), there are a large number of narrow and parallel lines with higher accumulation values which demonstrates the shortcoming of D8 for computing the spatial distribution of flow accumulation. This defect of SFD has been discussed by many researchers, such as Fairfield and Leymarie (1991), and Wilson and Gallant (2000). Compared with D8, MFD-Quinn has a much smoother spatial distribution of flow accumulation in which the state of flow in the low relief areas is much more reasonable (figure 6(b)).

The spatial distribution of flow accumulation by MFD-md is intergradation between that shown by D8 and MFD-Quinn. MFD-md has a similar result to MFD-Quinn where the main difference is in the areas of hillslope (figure 6(b) and figure (c)). Figure 7 shows a map of difference between figure 6(c) (MFD-md) and figure 6(b) (MFD-Quinn). The negative values in figure 7 mean that the flow accumulation value from MFD-md is smaller than that from MFD-Quinn (i.e. MFD-md minus MFD-Quinn). The positions with a large negative difference (i.e. the red region in figure 7) are mostly areas next to the v-shaped channels where slope gradients are high. This is because the fixed flow-partition exponent used in MFD-Quinn treats convergent flow as divergent flow and unnecessarily spreads flow to the neighbouring pixels, thus producing higher flow accumulation values for these pixels. For the same reason, for the v-shaped valleys, as in this study area, the pixels at the valley centre receive a smaller flow accumulation based on MFD-Quinn because the flow draining into them has been spread into other neighbouring pixels. This explains the larger positive differences in the valley centres between the two methods. The comparison of MFD-md, D8, and MFD-Quinn when applied to a real DEM shows that the proposed algorithm is advantageous in modelling the effect of local terrain on flow partitioning.

6. Conclusion and future work

Following on from Quinn *et al.* (1991)'s research, this paper proposes an adaptive scheme to varying the flow-partition exponent for MFD algorithms. Within this scheme, the flow-partition exponent is changed adaptively based on a local topographic attribute which not only reflects the terrain conditions but also controls the local drainage. Thus, the adaptive scheme can overcome the main problem of all MFD algorithms with a fixed p value.

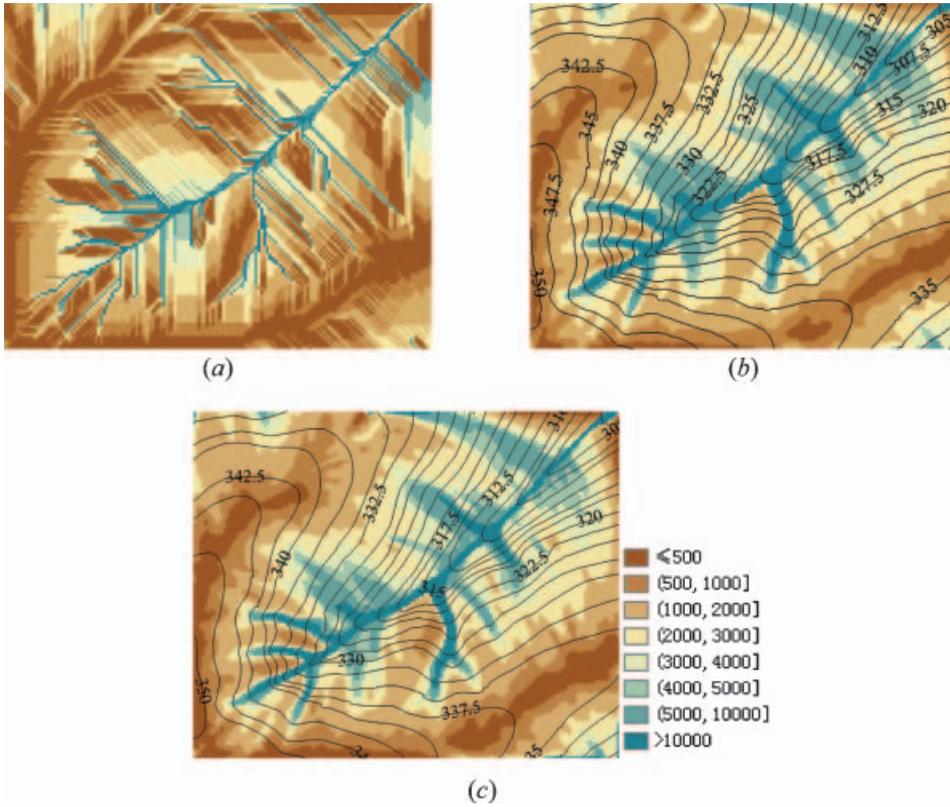


Figure 6. Spatial distribution of flow accumulation (m^2) computed by (a) D8, (b) MFD-Quinn, and (c) MFD-md. Contours with a 2.5-m interval of DEM are overlaid on the flow accumulation distribution.

This paper also presents the MFD-md algorithm, which is an implementation of the proposed adaptive flow-partition scheme. MFD-md selects the local maximum downslope gradient as the local topographic attribute in the adaptive scheme. Then,

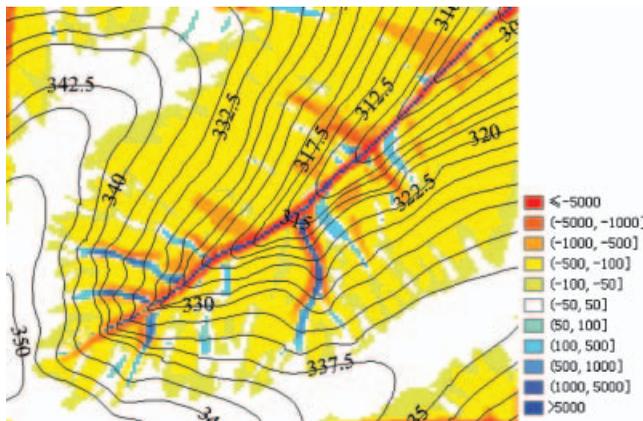


Figure 7. Difference in flow accumulations (m^2) computed by MFD-md and MFD-Quinn. Contours with a 2.5-m interval of DEM are overlaid on the figure.

MFD-md uses a function of the local maximum downslope gradient to vary the flow-partition exponent in response to local terrain conditions. Unlike some current MFDs with varying flow-partition exponents (e.g. Quinn *et al.* 1995), MFD-md does not need a spatial distribution of, or a threshold for, flow accumulation, which must be pre-computed with a flow-routing algorithm for determining the flow-partition exponent. Therefore, the MFD-md algorithm has a lower complexity and is independent of other flow-direction algorithms.

MFD-md is evaluated using four artificial surfaces. MFD-md has the lowest error rate in comparison with the D8 algorithm and the MFD-Quinn algorithm. This evaluation suggests that the MFD-md can adaptively accommodate the effects of terrain conditions on flow distribution. The real-world case further suggests that the spatial distribution of computed flow accumulation by MFD-md is more accurate than the result of D8 and is similar to the result of Quinn *et al.* (1991)'s MFD algorithm. Further analysis of valley regions shows that MFD-md is more adaptive to the terrain conditions than MFD-Quinn.

The adaptive scheme presented in this paper provides a flexible framework for varying flow-partition exponent for MFD. During the implementation of the adaptive scheme, the selection of both threshold of parameters and local topographic attributes will deduce different models for local flow partitioning. MFD with a fixed p value can be considered as a special case of this adaptive scheme. For example, MFD-Quinn is the case when the adaptive scheme takes $p_l = p_u = 1$ in equation (3). In future work, the impact of p_u , p_l , e_{\min} , and e_{\max} on MFD-md will be quantitatively studied in order to enhance the usability and robustness of this MFD-md algorithm. On the other hand, although the maximum downslope gradient might be the most important local topographic attribute controlling local drainage, it is not the only one. Other local topographic attributes, such as plan curvature and profile curvature, should be considered together with the maximum downslope gradient to model varying flow-partition exponent.

Acknowledgements

This study was funded through support from a grant (Project Number: 40501056) from National Natural Science Foundation of China, the 'Hundred Talents' Program of Chinese Academy of Sciences, and International Partnership Project 'Human Activities and Ecosystem Changes' (Project Number: CXTD-Z2005-1) of Chinese Academy of Sciences. Support from the University of Wisconsin-Madison is also appreciated.

References

- BERTOLO, F., 2000, *Catchment Delineation and Characterization: A Review (Tech. Rep. EUR 19563 EN*, Joint Res. Cent. Eur. Comm., Ispra, Italy, April. Available online at: (<http://agrienv.jrc.it/publications/pdfs/CatchRev.pdf> (accessed 01 February 2006).
- BEVEN, K.J. and KIRKBY, N.J., 1979, A physically based variable contributing area model of basin hydrology. *Hydrological Sciences Bulletin*, **24**, pp. 43–69.
- CHIRICO, G.B., WESTERN, A.W., GRAYSON, R.B. and BLOSCHL, G., 2005, On the definition of the flow width for calculating specific catchment area patterns from gridded elevation data. *Hydrological Processes*, **19**, pp. 2539–2556.
- COSTA-CABRAL, M.C. and BURGESS, S.J., 1994, Digital elevation model networks (DEMON): A model of flow over hillslopes for computation of contributing and dispersal areas. *Water Resources Research*, **30**, pp. 1681–1692.

- FAIRFIELD, J. and LEYMARIE, P., 1991, Drainage networks from grid digital elevation models. *Water Resources Research*, **27**, pp. 709–717.
- FREEMAN, T.G., 1991, Calculating catchment area with divergent flow based on a regular grid. *Computers & Geosciences*, **17**, pp. 413–422.
- GÜNTNER, A., SEIBERT, J. and UHLENBROOK, S., 2004, Modeling spatial patterns of saturated areas: An evaluation of different terrain indices. *Water Resources Research*, **40**, W05114, doi:10.1029/2003WR002864.
- HJERDT, K., McDONNELL, J., SEIBERT, J. and RODHE, A., 2004, A new topographic index to quantify downslope controls on local drainage. *Water Resources Research*, **40**, W05602, doi:10.1029/2004WR003130.
- HOLMGREN, P., 1994, Multiple flow direction algorithms for runoff modeling in grid-based elevation models: An empirical evaluation. *Hydrological Processes*, **8**, pp. 327–334.
- JENSON, S.K. and DOMINGUE, J.O., 1988, Extracting topographic structure from digital elevation data for geographic information system analysis. *Photogrammetric Engineering and Remote Sensing*, **54**, pp. 1953–1600.
- KIM, S. and LEE, H., 2004, A digital elevation analysis: a spatially distributed flow apportioning algorithm. *Hydrological Processes*, **18**, pp. 1777–1794.
- KONG, F.Z. and RUI, X.F., 2003, New method for treating flat areas and closed depressions in the digital elevation models. *Advances in Water Science*, **14**, pp. 290–294 (in Chinese).
- LEA, N.L., 1992, An aspect driven kinematic routing algorithm. In *Overland Flow: Hydraulics and Erosion Mechanics*, A.J. Parsons and A.D. Abrahams (Eds), pp. 393–407 (London: UCL Press).
- MARTZ, L.W. and DE JONG, E., 1988, CATCH: A FORTRAN program for measuring catchment area from digital elevation models. *Computers & Geosciences*, **14**, pp. 627–640.
- MARTZ, L.W. and GARBRECHT, J., 1998, The treatment of flat areas and depressions in automated drainage analysis of raster digital elevation models. *Hydrological Processes*, **12**, pp. 843–855.
- MOORE, I.D., GRAYSON, R.B. and LADSON, A.R., 1991, Digital terrain modeling: A review of hydrological, geomorphological, and biological applications. *Hydrological Processes*, **5**, pp. 3–30.
- MOORE, I.D., LEWIS, A. and GALLANT, J.C., 1993a, Terrain attributes: estimation methods and scale effects. In *Modelling Change in Environmental Systems*, A.J. Jakeman, M.B. Beck and M.J. McAleer, pp. 189–214 (New York: Wiley).
- MOORE, I.D., TURNER, A.K., WILSON, J.P., JENSEN, S.K. and BAND, L.B., 1993b, GIS and land surface-subsurface process modeling. In *Geographic Information Systems and Environmental Modeling*, M.F. Goodchild, B.O. Parks and L.T. Steyaert (Eds), pp. 196–230 (Oxford: Oxford University Press).
- O'CALLAGHAN, J.F. and MARK, D.M., 1984, The extraction of drainage networks from digital elevation data. *Computer Vision, Graphics, and Image Processing*, **28**, pp. 323–344.
- O'LOUGHLIN, E.M., 1986, Prediction of surface saturation zones in natural catchments by topographic analysis. *Water Resources Research*, **22**, pp. 794–804.
- PAN, F., PETERS-LIDARD, C., SALE, M. and KING, A., 2004, A comparison of geographical information system-based algorithms for computing the TOPMODEL topographic index. *Water Resources Research*, **40**, W06303, doi:10.1029/2004WR003069.
- PILESJÖ, P. and ZHOU, Q., 1997, Theoretical estimation of flow accumulation from a grid-based digital elevation model. In *Proceedings of GIS AM/FM ASIA'97 and Geoinformatics'97 Conference* (Taipei), pp. 447–456.
- PILESJÖ, P., ZHOU, Q. and HARRIE, L., 1998, Estimating flow distribution over Digital Elevation Models using a form-based algorithm. *Geographic Information Sciences*, **4**, pp. 44–51. Available online at: <http://www.lantm.lth.se/personal/lars/jgis98.pdf> (accessed 01 October 2005).

- QUINN, P., BEVEN, K., CHEVALIER, P. and PLANCHON, O., 1991, The prediction of hillslope flow paths for distributed hydrological modeling using digital terrain models. *Hydrological Processes*, **5**, pp. 59–79.
- QUINN, P., BEVEN, K.J. and LAMB, R., 1995, The $\ln(\alpha/\tan\beta)$ index: how to calculate it and how to use it within the TOPMODEL framework. *Hydrological Processes*, **9**, pp. 161–182.
- TARBOTON, D.G., 1997, A new method for the determination of flow directions and upslope areas in grid digital elevation models. *Water Resources Research*, **33**, pp. 309–319.
- WILSON, J.P. and GALLANT, J.C., 2000, Primary topographic attributes. In *Terrain Analysis: Principles and Applications*, J.P. Wilson and J.C. Gallant (Eds), pp. 51–85 (New York: Wiley).
- WOLOCK, D.M. and MCCABE, G.J., 1995, Comparison of single and multiple flow direction algorithms for computing topographic parameters. *Water Resources Research*, **31**, pp. 1315–1324.
- ZHOU, Q. and LIU, X., 2002, Error assessment of grid-based flow routing algorithms used in hydrological models. *International Journal of Geographical Information Science*, **16**, pp. 819–842.